

# Why Indexing Works

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## Abstract

We develop a simple stock selection model to explain why active equity managers tend to underperform a benchmark index. We motivate our model with the empirical observation that the best performing stocks in a broad market index perform much better than the other stocks in the index. While randomly selecting a subset of securities from the index increases the chance of outperforming the index, it also increases the chance of underperforming the index, with the frequency of underperformance being larger than the frequency of overperformance. The relative likelihood of underperformance by investors choosing active management likely is much more important than the loss to those same investors of the higher fees for active management relative to passive index investing. Thus, the stakes for finding the best active managers may be larger than previously assumed.

**Key words:** Indexing, Passive Management, Active Management

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# 1 Introduction

The tendency of active equity managers to underperform their benchmark index (*e.g.*, Lakonishok, Shleifer, and Vishny (1992), Gruber (1996)) is something of a mystery. It is one thing for active equity managers to fail *to beat* the benchmark index, since that may imply only a lack of skill to do better than random selection. It is quite another to find that most active equity managers fail *to keep up with* the benchmark index, since that implies that active equity managers are doing something that systematically leads to underperformance.

We develop a simple stock selection model that builds on the underemphasized empirical fact that the best performing stocks in a broad index perform much better than the other stocks in the index, so that average index returns depend heavily on the relatively small set of winners (*e.g.*, J.P. Morgan (2014)). In our model, randomly selecting a small subset of securities from the index maximizes the chance of outperforming the index - the allure of active equity management - but it also maximizes the chance of underperforming the index, with the chance of underperformance being larger than the chance of overperformance. To illustrate the idea, consider an index of five securities, four of which (though it is unknown which) will return 10% over the relevant period and one of which will return 50%. Suppose that active managers choose portfolios of one or two securities and that they equally-weight each investment. There are 15 possible one or two security “portfolios.” Of these 15, 10 will earn returns of 10%, because they will include only the 10% securities. Just 5 of the 15 portfolios will include the 50% winner, earning 30% if part of a two security portfolio and 50% if it is the single security in a one security portfolio. The mean average return for all possible actively-managed portfolios will be 18%, while the median actively-managed portfolio will earn 10%. The equally-weighted index of all 5 securities will earn 18%. Thus, in this example, the average active-management return will be the same as the index (see Sharpe (1991)), but two-thirds of the actively-managed portfolios will underperform the index because they will omit the 50% winner.

Our paper continues as follows. In Section 2, we develop our simple stock selection model. In Section 3, we present simulation results. Section 4 concludes.

## 2 A Simple Model of Stock Selection from an Index

We consider a benchmark index that contains  $N$  stocks  $S^i$ ,  $1 \leq i \leq N$ . Let the dynamics of stock  $S^i$  over time  $t \in [0, T]$  be given by a geometric Brownian motion

$$\frac{dS_{t+1}^i}{S_t^i} = \mu_i dt + \sigma dW_t,$$

where for simplicity we consider the volatility  $\sigma > 0$  to be constant for all stocks. We assume that stock drifts are distributed  $\mu_i \sim N(\hat{\mu}, \hat{\sigma}^2)$ , which generates a small number of extreme

winners, a small number of extreme losers, and a large number of stocks with drifts centered around  $\hat{\mu}$  with standard deviation  $\hat{\sigma} > 0$ . While our model implies unpriced covariance among securities and a lack of learning, much theory and evidence suggests that the learning problem is too difficult over the lifetimes of most investors to pay much attention to that modeling limitation (*e.g.*, Merton (1980), Jobert, Platania, and Rogers (2006)).<sup>1</sup>

For simplicity, we assume that individual stocks maintain their drift  $\mu_i$  over the time period  $t \in [0, T]$ . We also assume that individual stocks have a starting value  $S_0^i = 1$  for all stocks.

If at time  $t = 0$  we pick a stock  $S_0^i$  at random, then our time  $T$  value follows

$$S_T^i \sim e^{\hat{\mu}T - \frac{1}{2}\sigma^2T + \sqrt{\sigma^2T + \hat{\sigma}^2T^2}Z},$$

where  $Z \sim N(0, 1)$ , provided we assume  $\mu_i$  and  $W_T$  are independent.

We define

$$I_t^N = \frac{1}{N} \sum_{i=1}^N S_t^i,$$

which corresponds to a capital weighted index of  $N$  stocks. By the Central Limit Theorem,

$$I_T^N \approx \mathbb{E}[S_T^i] = e^{\hat{\mu}T + \frac{1}{2}\hat{\sigma}^2T^2} \quad (1)$$

for large  $N$ .

Two observations are apparent. First, the cumulative return of a stock picked randomly at time  $t = 0$  follows a log- $N(\hat{\mu}T - \frac{1}{2}\sigma^2T, \sigma^2T + \hat{\sigma}^2T^2)$  distribution. The variance component  $\hat{\sigma}^2T^2$ , which indicates the over-proportional profit a continuously compounded winner will bring relative to the loss incurred by a loser. That is, the distribution is heavily positively skewed with a mean of  $e^{\hat{\mu}T + \frac{1}{2}\hat{\sigma}^2T^2}$ . Second, the median of the stock distribution is given by  $e^{\hat{\mu}T - \frac{1}{2}\sigma^2T}$ , so that over time  $T$  more than half of all stocks in the index will underperform the index return  $I_T^N$  by a factor of  $e^{\frac{1}{2}\sigma^2T + \frac{1}{2}\hat{\sigma}^2T^2}$ .

### 3 Simulation Results

We assume a median index return of 10% and an expected index return of 50% over the considered period  $T$ . We take  $\sigma = 20\%$  as a generic annual stock volatility. We choose  $T = 5$  (five years),  $\hat{\mu} = (\log 1.1 + \frac{1}{2}0.2^2 \cdot 5)/5 \approx 4\%$ , and  $\hat{\sigma} = \sqrt{2 \log 1.5 - 0.04 \cdot 5 \cdot 2}/5 \approx 13\%$ . We

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<sup>1</sup>In one study of stock market fluctuations, Barsky and DeLong (1993) discuss the problem of estimating a particular parameter for an assumed dividend process, noting that a Bayesian updater might not be shifted significantly from his prior after 120 years of data and that “[e]ven if we were lucky and could precisely estimate [the parameter], no investor in 1870 or 1929-lacking the data that we possess-had any chance of doing so.”

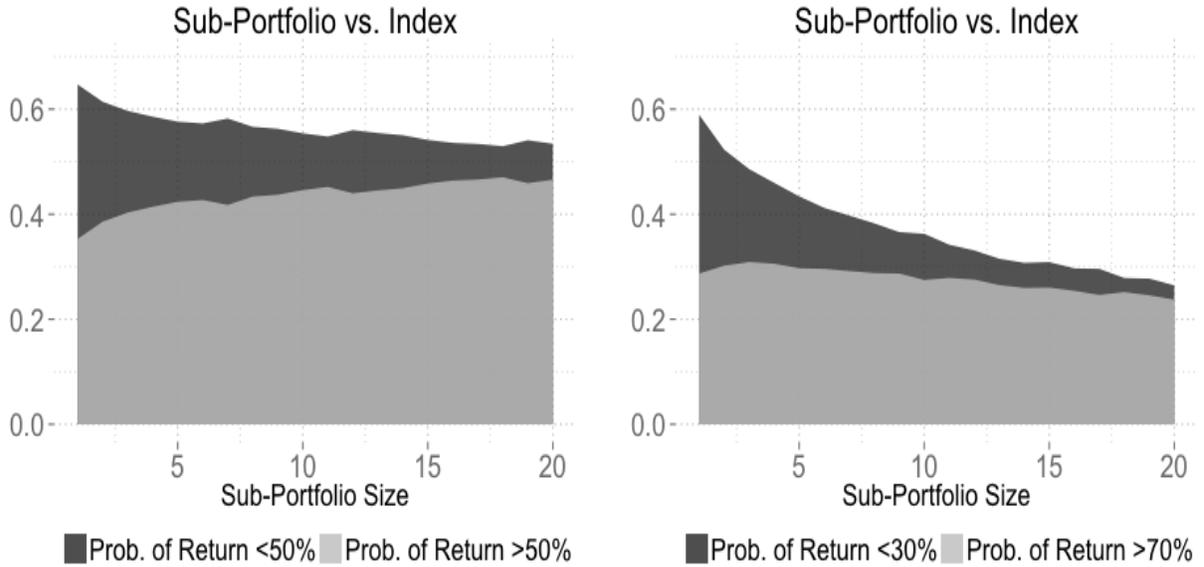


Figure 1: *On the left, overlapping frequencies of over- and underperformance relative to index average return of 50%. On the right, overlapping frequencies of 20% over- and underperformance relative to index average return 50%. While random selection of small sub-portfolios has the greatest probability of getting overperformance, it also risks a relatively high probability of underperformance. The risk of substantial index underperformance always dominates the chance of substantial index outperformance and is greatest for small portfolios.*

show the frequency of exceeding or falling short of the expected five year, 500-stock index return  $\mathbb{E}I_{T=5}^{N=500} - 1 \approx 50\%$  when creating sub-portfolios of different sizes (each computed based on a Monte Carlo simulation with 10,000 samples). Figure 1 left shows the frequency with which randomly selected portfolios of a given size overperform (5 year return greater than 50%) and underperform (5 year return less than 50%) the expected return for all 500 stocks. Figure 1 right shows the frequency with which randomly selected portfolios of a given size overperform (5 year return greater than 70%) and underperform (5 year return less than 30%) using more extreme thresholds for over- and underperformance.

The risk of substantial index underperformance always dominates the chance of substantial index outperformance, with the difference being greater the smaller the size of the selected sub-portfolios. It is far more likely that a randomly selected subset of the 500 stocks will underperform than overperform, because average index performance depends on the inclusion of the extreme winners that often are missed in sub-portfolios.

## 4 Conclusion

Researchers have focused on the costs of active management as being primarily the fees paid for active management (*e.g.*, French 2008). Our results suggest that the much higher cost of active management may be the inherently high chance of underperformance that comes with attempts to select stocks, since stock selection disproportionately increases the chance of underperformance relative to the chance of overperformance. To the extent that those allocating assets have assumed that the only cost of active investing above indexing is the cost of the active manager in fees, it may be time to revisit that assumption. The stakes for identifying the best active managers may be higher than previously thought.

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