BEYOND THE STATUS QUO: A CRITICAL ASSESSMENT OF LIFECYCLE INVESTMENT ADVICE

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Abstract

We challenge two tenets of lifecycle investing: (i) diversify across stocks and bonds and (ii) reduce equity allocations with age. We estimate optimal age-based weights in a lifecycle model that nonparametrically preserves time-series and cross-sectional dependencies in asset class returns. The optimal weights remain near one-third domestic stocks and two-thirds international stocks regardless of age—with no material fixed income allocation. This strategy dominates conventional stock-bond strategies in building wealth, preserving capital, and generating bequests. Our investors prefer diversifying with international stocks, not bonds. Target-date fund investors need 63% more pre-retirement savings to match the optimal strategy's expected retirement utility.

JEL classifications: C15, D14, G11, G17, G51

Key words: Lifecycle asset allocation, retirement savings, target-date funds, long-horizon returns

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1 Introduction

Every year, Americans contribute about 5% of their total employee compensation to defined contribution (DC) pension plans, with contributions of $621 billion in 2022 alone.[1] They then face a question that determines their financial fate: How should I invest my savings? Many consult financial advisors. These professionals impart two central tenets of lifecycle investing—people should diversify across stocks and bonds and the young should invest more heavily in stocks than the old—perhaps having learned them from investments textbooks [e.g., Bodie, Kane, and Marcus (2024)] or CFA study materials [e.g., Blanchett, Cordell, Finke, and Idzorek (2023)]. Self-directed savers who read a popular book by Dave Ramsey, Suze Orman, or Tony Robbins receive similar advice [Choi (2022)]. Academics may study the literature on lifecycle investing and reach the same conclusions [e.g., Viceira (2001); Campbell and Viceira (2002); and Cocco, Gomes, and Maenhout (2005)]. A great many others are uninterested or overwhelmed, so they invest in the default option of their employer-sponsored retirement plan. To safeguard these investors, the Pension Protection Act of 2006 (PPA) created safe harbors for employer DC plans. The most popular Qualified Default Investment Alternatives (QDIAs) are portfolios that provide "long-term appreciation and capital preservation through a mix of equity and fixed income exposures based on the participant's age" [29 CFR § 2550.404c-5(e)(4)(i)].[2] As such, regulators rely on "generally accepted investment theories" [29 CFR § 2550.404c-5(e)(4)] that mirror the two principles to define QDIAs. In summary, these two tenets of investment advice—split investments across stocks and bonds and invest more in stocks while young than while old—are close to being uniformly given and universally followed.

In this paper, we challenge these principles of lifecycle investing. We consider a US couple who optimizes expected utility over real retirement consumption and bequest within a lifecycle model with labor income risk, Social Security income, and longevity risk. Our innovation relative to prior work lies in the modeling of investment opportunities. We depart from conventional modeling practice by adopting a nonparametric block bootstrap approach that avoids ex-ante selection of which return features matter for portfolio choice and parametric assumptions about their functional forms. Our block bootstrap generates forward-looking holding-period returns by resampling long sequences of consecutive historical returns. These sequences are drawn from a comprehensive dataset of returns on domestic stocks,

[1]The total DC plan contribution is from the 2024 Private Pension Plan Bulletin from the Department of Labor. The 5% figure divides $621 billion in 2022 DC contributions by $13,437 billion in 2022 compensation of employees from Table 2.1 of the national income and product accounts (NIPA) from the Bureau of Economic Analysis.

[2]See https://www.law.cornell.edu/cfr/text/29/2550.404c-5. Vanguard (2024) reports that target-date funds, which are QDIAs under 29 CFR § 2550.404c-5(e)(4)(i), make up 98% of the QDIAs in DC plans.

international stocks, government bonds, and government bills spanning over 2,600 years of country-month data from 39 developed countries.[3] As such, our modeling approach preserves the time-series and cross-sectional properties of stock and bond returns while capturing the latent dynamics of the investment opportunity set. Across a range of model specifications, the household's optimal weights are approximately one-third domestic stocks and two-thirds international stocks at all ages—with virtually no fixed income allocation. The optimal strategy stands in sharp contrast to the status quo investment advice favoring age-based stock-bond portfolios.

We deliberately design our lifecycle model and optimization framework to circumvent the intractable challenge of parametric return modeling and to preserve the full complexity of returns relevant for optimal portfolio choice. Any realistic characterization of investment opportunities requires an infeasibly large number of parametric choices to model the levels and time variation in means, standard deviations, higher-order moments, correlations, and higher-order cross-moments. These parametric assumptions ultimately drive conclusions about optimal portfolio choice [e.g., Michaud (1989); Brandt (1999); Garlappi, Uppal, and Wang (2007); Wachter (2010); and Pástor and Stambaugh (2012)], such that omissions or inaccuracies can seriously undermine the analysis. Our nonparametric bootstrap method addresses these issues by preserving the underlying time-series and cross-sectional patterns in realized returns. To model the household's optimization problem, we incorporate bounded rationality using the sparse max optimization framework of Gabaix (2014). This approach recognizes that households, like researchers, face significant challenges in modeling returns. Our baseline household optimizes by choosing a sequence of age-based portfolio weights. The age-based portfolio choice framework also creates a natural parallel with status quo target-date funds (TDFs). TDFs condition their stock-bond portfolios on investor age, an approach that both complies with QDIA regulations and is easily implementable within employer-sponsored plans. As such, our findings are directly informative about optimal TDF design.

In the base case of our lifecycle model, the couple chooses an age-based allocation across domestic stocks, international stocks, bonds, and bills. We impose constraints—no leverage and non-negative weights—that reflect reality for most retirement savers. The optimal weights are remarkably stable as a function of age, with optimal allocations of approximately one-third in domestic stocks and two-thirds in international stocks throughout the lifecycle. Although including international stocks in the

[3] We consider multiple risky assets but no risk-free asset, similar to Campbell and Viceira (2002, 2005); Campbell, Chan, and Viceira (2003); Sangvinatsos and Wachter (2005); Hoevenaars, Molenaar, Schotman, and Steenkamp (2008); Koijen, Nijman, and Werker (2010); and Duarte, Fonseca, Goodman, and Parker (2024). A significant majority of studies include a risk-free asset, either with a single risky asset [e.g., Merton (1969); Samuelson (1969); Viceira (2001); Cocco, Gomes, and Maenhout (2005); Pástor and Stambaugh (2012); Dahlquist, Setty, and Vestman (2018); Gomes, Michaelides, and Zhang (2022); Gomes and Smirnova (2023); and Choukhmane and de Silva (2024)] or with multiple risky assets [e.g., Merton (1971); Brennan, Schwartz, and Lagnado (1997); Lynch (2001); Lynch and Tan (2010); and Catherine (2022)].

investment opportunity set is rare in the lifecycle investing literature, the large weight on this asset class underscores the importance of allowing for international diversification.[4]

The only exception to an all-equity approach occurs briefly at retirement: bills receive a 27% weight at age 65, but this weight shrinks to 7% by age 68 and 0% by age 70. We demonstrate that this allocation to bills is driven by a desire for tactical cash reserves under a rigid retirement withdrawal rule [i.e., the common 4% rule of Bengen (1994)]. With a more flexible withdrawal approach, the optimal strategy remains all equity throughout the lifetime. The tactical cash allocation also does not provide meaningful economic benefits compared with maintaining full equity exposure.

We compare the optimal age-based strategy with two QDIA benchmarks: (i) a balanced strategy with 60% domestic stocks and 40% bonds and (ii) a representative TDF. To achieve the same expected utility from retirement consumption and bequest as a couple investing in the optimal strategy and saving 10.00% of labor income, a couple using the balanced strategy must save 19.44% of income (i.e., nearly twice as much). The TDF performs moderately better, but still requires a substantially higher savings rate of 16.27% (i.e., 63% more) to match the expected utility of the optimal age-based strategy. These findings highlight the substantial welfare costs of following conventional investment advice.

We examine the determinants of expected utility by studying four retiree outcomes: wealth at retirement, retirement income, capital preservation, and bequest at death. The optimal age-based strategy dominates the two QDIAs in long-term appreciation, with 50% more retirement wealth on average than the balanced strategy and 39% more than the TDF. Although this strong performance in capital appreciation may be expected, our simulations also reveal a more striking result: the optimal strategy compares favorably with the QDIAs in capital preservation even with its dependence on high equity exposure throughout retirement. Under the 4% withdrawal rule, a couple using the balanced strategy has a 16.9% probability of running out of wealth before death. The TDF is even worse at 19.7%. In comparison, the probability for the optimal age-based strategy is low at 6.7%. Finally, the optimal strategy produces average bequests well over twice as large as the benchmarks. Overall, the optimal age-based portfolio beats the two QDIAs across the board in achieving the PPA goals of long-term appreciation and capital preservation.

We emphasize that we do not make any theoretical arguments that age does not matter for asset

[4]Baxter and Jermann (1997) consider international diversification in the context of non-tradeable human capital, concluding that investors should short domestic markets to hedge labor income risk. Michaelides (2003); Hnatkovska (2010); Coeurdacier and Rey (2013); and Bretscher, Julliard, and Rosa (2016) study these hedging motives in lifecycle asset allocation models featuring foreign stocks, concentrating on the roles of frictions, incomplete markets, and the strength of the relation between human capital and domestic asset returns in rationalizing home bias. Several researchers examine international diversification in settings without lifecycle features [e.g., Solnik (1974), Jorion (1985), Eun and Resnick (1988, 1994), French and Poterba (1991), and Ang and Bekaert (2002, 2004)].

allocation, that diversification is unimportant, or that bonds are categorically unsuitable. Rather, our empirical finding of persistently high equity allocations throughout the lifecycle stems from important modeling features: the preservation of time-series and cross-sectional return properties, the availability of international diversification, and the household's emphasis on real returns. Under these conditions, the couples are simply choosing to diversify throughout the lifecycle using international stocks rather than bonds.

This choice reflects the properties of long-horizon returns uncovered by our nonparametric bootstrap. To illustrate, Table I reports the annualized mean and standard deviation of returns on bonds and international stocks (Panel A); the variance ratios at horizons of one, ten, 20, and 30 years calculated as in Poterba and Summers (1988) (Panel B); and the correlations of log returns and log inflation (Panel C). Bonds offer modest average real returns (0.95% annually) compared with international stocks (7.03%), requiring substantial diversification benefits to justify their inclusion in an optimal lifecycle portfolio. At the one-month horizon, bonds appear less risky with lower standard deviation (9.51% versus 23.26%) and lower correlation with domestic stocks (0.21 versus 0.33). At the 30-year horizon, the picture changes. Bonds' per-year variance increases to 2.30 times the one-year variance, but international stocks' decreases to 0.75 times. The correlation of bonds with domestic stocks rises to 0.45, whereas international stocks maintain a steady correlation.[5] International stocks better preserve real buying power (correlation with inflation of −0.01), as bonds suffer during inflationary periods (correlation of −0.78). In sum, bonds ultimately seem unattractive for long-horizon investors. They have low returns, high long-term variance, high long-term correlation with domestic stocks, and high exposure to inflationary periods.

Given the evidence against bonds, what gives rise to the conventional wisdom that favors large bond allocations for retirees? Two methodological choices—preserving time-series dependencies via block bootstrap and including international stocks—appear to explain the disconnect. Much of the lifecycle investing literature relies on short-term return moments and focuses on domestic markets.[6] To examine the consequences of these design choices, we create an analogous specification within our framework using an independent and identically distributed (IID) bootstrap and a domestic-only asset set (i.e., domestic stocks, bonds, and bills). This simplified, convention-inspired approach yields

[5]Also see Campbell and Viceira (2002, 2005); Bali, Demirtas, Levy, and Wolf (2009); and Siegel (2014) for evidence on the risk of bonds over extended holding periods.

[6]It is possible to interpret the "stocks" asset class in previous lifecycle studies as representing a mix of domestic and international stock markets. If so, these studies make implicit assumptions about the relative weights across countries as well as the interdependencies of domestic and foreign stock returns, inflation, and exchange rates. By separately modeling domestic and international stocks, we allow investors to choose optimal weights and capture the rich patterns in returns, inflation, and exchange rates across markets.

markedly greater reliance on fixed income: at age 65, for example, the optimal portfolio comprises 38% domestic stocks, 14% bonds, and 48% bills. Varying the design reveals two important patterns. First, bond allocations are lower with a block bootstrap compared with an IID bootstrap, reflecting bonds' poor long-horizon return properties from Table I. Second, adding international stocks to the investment opportunity set reduces the weight in fixed income, consistent with international stocks' favorable long-horizon properties. In reality, returns do have time-series dependencies and households can invest internationally, so incorporating these features is both prudent and crucial for optimal portfolio choice.

Our conclusions prove robust across diverse model specifications. We consider the simpler optimization problem of choosing fixed weights throughout the lifetime. The optimal fixed-weight policy of 33% domestic stocks, 67% international stocks, 0% bonds, and 0% bills closely mirrors the optimal age-based policy and achieves virtually the same expected utility, with a 10.07% equivalent savings rate relative to the baseline of 10.00%. Our results are also robust to the sample period (i.e., restricting to post-World War II), bootstrap design, risk aversion, strength of the bequest motive, retirement withdrawal strategy, retirement age, contribution rate, and household type (e.g., single versus couple). Our findings are not driven by data from the US, small countries, or small stock markets.

Our base case is a median investor in Guvenen, Karahan, Ozkan, and Song's (2021) model of stochastic labor income. Alternative household types with a range of initial income and human capital parameters choose strategies that mirror the base case. We also introduce correlation between labor income and domestic stock returns using a Gaussian copula; investors adjust their allocation across domestic and international stocks, but they do not buy bonds.

We assume the retirement contribution rate and retirement date are exogenous in the base case, but explore two extensions: replacing the constant contribution rate with age- and income-based rates from Parker, Schoar, Cole, and Simester (2023), and allowing couples to choose optimal retirement timing based on income, wealth level, and Social Security benefits. These features are important for investors' utility but not for portfolio choice in our setting, leaving the optimal policy unchanged. Allowing market timing based on the domestic price-dividend ratio yields an all-equity strategy, except for a modest 9% weight in bonds in the highest-valuation quintile. Finally, we allow borrowing up to 100% of wealth at the bill yield plus a modest margin spread. The couple borrows 55% of wealth and effectively leverages the optimal fixed-weight strategy by allocating 34% to domestic stocks and 66% to international stocks. Overall, the optimality of investing in equity throughout the lifetime with minimal fixed income exposure is robust across specifications.

We contribute to the recent normative literature on the optimal design of lifecycle investment strategies [e.g., Michaelides and Zhang (2017, 2022); Dahlquist, Setty, and Vestman (2018); Kraft, Munk, and Weiss (2019); Gomes, Michaelides, and Zhang (2022); and Duarte, Fonseca, Goodman, and Parker (2024)]. Our primary contribution to the lifecycle literature is our modeling of the investment opportunity set. Several classic studies assume constant investment opportunities with returns that are normally or lognormally distributed [e.g., Merton (1969); Viceira (2001); Cocco, Gomes, and Maenhout (2005); and Gomes and Michaelides (2005)]. Many other studies consider particular aspects of dynamic investment opportunities or non-normalities in returns, finding that optimal lifecycle allocations are affected by time-varying expected returns, time-varying return variance, and skewness in returns.[7] Dynamic programming methods are commonly used to find optimal portfolio weights in the Markov decision processes (MDPs) studied in this literature, and researchers must make explicit parametric assumptions about investment opportunities to satisfy technical conditions [e.g., Stokey, Lucas, Jr., and Prescott (1989)]. In contrast, we adopt a Monte Carlo optimization method, which is a reinforcement learning approach to MDPs that finds an approximately optimal policy when transition probabilities can be simulated rather than explicitly modeled. Our block bootstrap simulation preserves empirically relevant features of investment opportunities. This approach avoids both parametric model specification and ex-ante assumptions about which return characteristics drive optimal portfolio choice.

Although studies with time-varying investment opportunities provide important insights into optimal intertemporal hedging demands, Cochrane's (2014, 2022) alternative perspective for long-term investors focuses on asset payoffs rather than the modeling of state-dependent investment opportunities. Cochrane (2014) states, "the hedging demands emphasized by the portfolio approach are really means to an end—an optimal consumption stream—rather than the end itself." Our lifecycle optimization approach embodies this perspective by focusing on the long-horizon asset payoffs that support retirement consumption and bequest. With our method, the expected utility for a given strategy reflects complex return dynamics that are implicit in holding-period returns without an explicit model of these complexities. Investors who approach the lifecycle problem with a focus on long-term payoffs optimally choose internationally diversified equity portfolios throughout their lifetimes.

[7] See, e.g., Campbell and Viceira (1999); Barberis (2000); Lynch (2001); Wachter (2002); Campbell, Chan, and Viceira (2003); Pástor and Stambaugh (2012); Michaelides and Zhang (2017, 2022); and Gomes, Michaelides, and Zhang (2022) for evidence on time-varying expected returns; Lynch and Balduzzi (2000) and Chacko and Viceira (2005) for time-varying variance; and Fagereng, Gottlieb, and Guiso (2017); Catherine (2022); Shen (2024); and Bonaparte, Korniotis, Kumar, Michaelides, and Zhang (2025) for skewness.

2 The status quo in lifecycle investing

In seminal studies, Merton (1969) and Samuelson (1969) provide a baseline for lifecycle asset allocation. They demonstrate that investors have constant optimal allocations to a risky asset and a risk-free asset under the conditions that human capital is tradeable and investment opportunities are constant. Subsequent studies addressing the investors' lifecycle problem relax these assumptions.[8]

Human capital, as a dominant asset for many working-age investors, is the focus of much of the literature on lifecycle portfolio choice. With complete markets, investors capitalize labor income and optimal asset allocation is unaffected [Merton (1971)]. With incomplete markets (e.g., non-tradeable, non-insurable labor income and borrowing constraints), in contrast, Cocco, Gomes, and Maenhout (2005) demonstrate that investors optimally choose age-based allocations.[9] If labor income risk is idiosyncratic, human capital substitutes for the risk-free asset in the optimal portfolio and the young hold more in stocks than the old.[10] Bodie, Merton, and Samuelson (1992) consider endogenous labor supply and retirement with utility over consumption and leisure, finding that labor supply flexibility increases optimal financial portfolio risk. Because the young have more labor flexibility than the old, the young should hold more in stocks.[11] Reinforcing these human capital effects, mean reversion in stock returns also makes stocks more attractive for young investors with long horizons [e.g., Barberis (2000), Wachter (2002), and Siegel (2014)].

Regulation stemming from the PPA builds upon these findings to favor an "investment fund product or model portfolio that applies *generally accepted investment theories* [emphasis added], is diversified so as to minimize the risk of large losses and that is designed to provide varying degrees of long-term appreciation and capital preservation through a mix of equity and fixed income exposures based on the participant's age, target retirement date (such as normal retirement age under the plan) or life expectancy" [29 CFR § 2550.404c-5(e)(4)(i)]. TDFs meet these design criteria by following age-based, stock-bond asset allocation policies. They adopt allocations with higher exposures to equities for younger investors and increase exposures to fixed income assets as the investors age. Figure 1 illustrates this design with

[8]We refer readers to excellent reviews by Campbell (2006); Gomes (2020); and Gomes, Haliassos, and Ramadorai (2021).

[9]See also Heaton and Lucas (1997), Koo (1998), and Viceira (2001) for treatments of non-insurable labor income risk in infinite-horizon models.

[10]Human capital can substitute for the risky asset with a systematic component in labor income [e.g., Viceira (2001); Campbell and Viceira (2002); Benzoni, Collin-Dufresne, and Goldstein (2007); and Lynch and Tan (2011)], such that the optimal risky allocation decreases in the presence of labor income if the correlation between labor income shocks and stock returns is sufficiently high.

[11]Farhi and Panageas (2007); Chai, Horneff, Maurer, and Mitchell (2011); and Hubener, Maurer, and Mitchell (2016), among others, also study the implications for portfolio choice of flexible labor supply and endogenous retirement. Other studies consider the effects of nonemployment on portfolio decisions [e.g., Bremus and Kuzin (2014); Fagereng, Guiso, and Pistaferri (2018); and Bagliano, Fugazza, and Nicodano (2019)].

the advertised unconditional glidepath weights in domestic stocks, international stocks, bonds, and bills from a TDF offered by a major investment firm.

TDFs have exploded in popularity since the passage of the PPA, with total assets under management (AUM) reaching $3.5 trillion at year-end 2023 [Parker, Schoar, and Sun (2023) and Pacholok (2024)]. The growth reflects the aggressive trend of plan sponsors' adoption of automatic enrollment features, the overwhelming tendency to select TDFs as default funds, and the propensity for participants to retain the default elections [e.g., Madrian and Shea (2001) and Mitchell and Utkus (2022)].[12] For a typical employee, the default option is a TDF that matches employee age to a target retirement date without consideration of other employee characteristics [Balduzzi and Reuter (2019)]. Across all participants in Vanguard plans, 83% use TDFs and 58% hold their entire account balance in a single TDF [Vanguard (2024)]. In short, age-based TDF strategies now represent the status quo for retirement saving.

3 Data

In this section, we describe the data on asset class returns used in our analyses. We take the perspective of a US couple saving for retirement. The investment horizons of retirement savers are very long, extending 75 years or beyond for young savers. The relatively short history of US financial markets poses a small sample problem given this setting, so we model forward-looking returns by examining the history of asset class returns from a broad cross section of developed economies. We follow Anarkulova, Cederburg, and O'Doherty (2022) in classifying countries as developed. The dataset includes monthly real returns for domestic stocks, international stocks, government bonds, and government bills for 39 developed countries. The data cover the period from 1890 to 2023, but the sample periods for individual countries vary based on data availability and the timing of economic development (i.e., a given country is included in the sample only for the period after it achieves developed status).

The starting point for constructing the dataset is the GFDatabase from Global Financial Data. For each sample country, the GFDatabase contains times series of total return indexes, price indexes, dividend-price ratios, and total market capitalization for stocks; yields for ten-year government bonds and short-term bills; consumer price indexes; and exchange rates. The internet appendix provides detailed

[12]As a default option, TDFs benefit investors who are inattentive, have behavioral biases, or lack financial literacy by offering diversification benefits and automatic reallocations. A large literature shows suboptimal investment decisions related to international diversification [Bekaert, Hoyem, Hu, and Ravina (2017)], asset allocation [Benartzi and Thaler (2001)], contribution levels [Lusardi and Mitchell (2007, 2011) and Goda, Levy, Manchester, Sojourner, and Tasoff (2020)], stock market participation [van Rooij, Lusardi, and Alessie (2011)], and account concentration in employer stock [Poterba (2003)]. Campbell (2016); Beshears, Choi, Laibson, and Madrian (2018); and Gomes, Haliassos, and Ramadorai (2021) provide comprehensive reviews of this evidence.

descriptions of the appropriate GFD data series for each country, alternative sources used to fill gaps in the GFDatabase, calculations of asset class returns, adjustments to these calculations for several periods surrounding major market disruptions (e.g., the closure of the New York Stock Exchange in 1914 at the onset of World War I and the Greek government bond default in 2012), and dataset validation.

The dataset is a balanced panel in the sense that each country-month has non-missing returns for domestic stocks, international stocks, bonds, and bills. The nominal returns for domestic stocks, bonds, and bills for a given country are measured in the local currency; the nominal returns are then converted to real returns using the local inflation rate. The nominal international stock returns for a given country are market-capitalization-weighted averages of the nominal returns for all non-domestic stock markets, with appropriate adjustments for changes in exchange rates. Analogous to the calculations for the other asset classes, the nominal international stock returns are converted to real returns based on local inflation.[13] As such, all asset class returns for a given country-month reflect the real investment outcomes of local investors in that month.

The broad sample of asset class returns allows for a more comprehensive characterization of potential investment outcomes relative to samples based on single countries (e.g., the US or the UK). Although single-country samples are commonly used to calibrate inputs for investment simulations, such samples contain few independent observations of long-horizon investment outcomes. Moreover, these samples suffer from both survivor bias [e.g., Brown, Goetzmann, and Ross (1995)] and easy data bias [e.g., Dimson, Marsh, and Staunton (2002)]. These biases can contribute to long historical periods over which realized performance exceeds ex ante expectations [e.g., Fama and French (2002) and Avdis and Wachter (2017)].[14]

In Table II, we list each sample country and the corresponding data coverage. Five countries—Denmark, France, Germany, the UK, and the US—are included in the sample over the full 1890 to 2023 period. The sample periods for the other countries are shorter owing to data availability and development classification status. Our data cover 91% of the potential country-months in the developed country sample. Table II also presents the geometric average real return and the standard deviation of real return for each combination of country and asset class. For the pooled sample of all 31,801 country-

[13] Anarkulova, Cederburg, and O'Doherty (2025) show that the value-weighted average of foreign stock market real returns and changes in real exchange rates each contribute about half of the overall volatility of the monthly international stock returns in our dataset.

[14] The dataset construction methods are designed to mitigate survivor bias and easy data bias. The sample inclusion dates for individual countries are based on ex ante measures of economic activity (e.g., the proportion of a country's labor force employed in the manufacturing and services sectors and the country's membership in global policy organizations like the Organisation for Economic Co-operation and Development), and we take significant steps to construct continuous monthly data series for each country.

month observations, the geometric average return is 0.37% for domestic stocks, 0.44% for international stocks, 0.04% for bonds, and −0.03% for bills (untabulated). Based on comparisons with the pooled sample, the average real returns in the US sample are higher for domestic stocks, bonds, and bills and lower for international stocks. But the US is not an extreme outlier relative to other countries for any of the four asset classes.[15]

4 Methods

The lifecycle portfolio choice problem of our household, like most problems in this literature, fits within the Markov decision process (MDP) framework. The goal in the MDP framework is to find the policy that maximizes expected reward through state-dependent actions. Dynamic programming is an important algorithm for MDP problems. It requires that the states and transition probability distributions are fully specified [e.g., Stokey, Lucas, and Prescott (1989)]. The standard approach in the lifecycle portfolio choice literature involves making simplifying assumptions, such as assuming that risk-free returns are known and constant, and that risky asset returns are either IID or follow a conditional distribution characterized by a small number of state variables. Such assumptions are mathematically convenient. Because the assumptions imply that the state variables and transition probability distributions are fully specified, the researcher can formulate a Bellman equation and apply a standard recursive dynamic programming approach to optimize expected utility.

In reality, the set of state variables required to describe transition probability distributions (i.e., conditional return distributions) is neither small nor known. Realistically modeling returns is extraordinarily difficult, leading Brandt (2010) to characterize return modeling as "without doubt the Achilles' heel of the traditional econometric approach" to asset allocation.[16] Undoubtedly, any set of state variables specified by an econometrician will be incomplete, such that the transition probability distributions are not fully specified. The disconnect between typical modeling assumptions and reality has two important implications for modeling and solving the household's problem.

[15]The standard deviation of bond returns for Germany is extreme relative to the estimates for other sample countries. This volatility is attributable to the well-known period of hyperinflation in Germany in the 1920s. Excluding all German data from our model has no material impact on the optimal asset allocation policies described in Section 5.

[16]The academic literature has proposed dozens of stock market return predictors [e.g., Goyal, Welch, and Zafirov (2024)] that capture distinct expected return patterns at short and long horizons [e.g., Cederburg, Johnson, and O'Doherty (2023) and Anarkulova, Cederburg, and Zhou (2025)]. Additional work has developed a host of methods for forecasting stock market volatility [e.g., Engle (1982) and Bollerslev (1986)] and many approaches to estimating time-varying tail risk in stocks [e.g., Kelly and Jiang (2014)]. The extensive set of state variables implied by this literature would cover only domestic stocks. Expanding the analysis to multiple asset classes introduces additional modeling complexities for currencies, inflation, interest rates, and both cross-asset and cross-country dependencies.

First, the household faces the same difficult modeling problem as does the econometrician. Assuming that the household knows the true conditional return distribution in such a complex environment is tenuous [e.g., Hansen (2007)]. Adding to the difficulties presented by the sheer dimensionality of the problem, it is costly for investors to acquire information [e.g., Huang and Liu (2007) and Nieuwerburgh and Veldkamp (2010)] and investors have limited information-processing capabilities [e.g., Sims (2003, 2006)]. These characteristics of the problem naturally lead to a design in which the household has bounded rationality. We therefore adopt the sparse max framework of Gabaix (2014), in which the household solves a simplified lifecycle problem by attending only to variables of first-order importance (e.g., age). This bounded rationality formulation is also consistent with prevailing industry practice, which relies on simple age-based allocation rules as implemented in TDFs.

Second, the standard dynamic programming approach is not applicable because the conditional return distributions are not fully specified. Reinforcement learning algorithms, in contrast, do not require *a priori* knowledge of transition probabilities and learn optimal policies through simulation rather than explicit modeling. Within this class, we adopt a Monte Carlo optimization method that finds approximately optimal solutions for MDPs when transition probabilities can be simulated rather than analytically specified. Our nonparametric block bootstrap approach generates asset class return sequences that preserve crucial time-series and cross-sectional dependencies. We then find approximately optimal sequences of portfolio weights with Monte Carlo by maximizing average household utility across draws. This approach allows complex return dynamics to influence optimal lifecycle strategies without requiring that these complexities be explicitly modeled. In sum, our methods are designed to preserve realistic return properties and directly confront the fact that conditional return distributions that match reality cannot be formally specified.

The remainder of this section details our lifecycle assumptions, presents the household's portfolio choice problem, and discusses our approaches for modeling uncertainty over labor income, investment returns, and longevity. Section 4.1 describes the lifecycle design. Section 4.2 defines and parameterizes household utility over retirement consumption and bequest and introduces our base case optimization problem. Section 4.3 presents the stochastic process for labor income during the working period and the design of Social Security income during the retirement period. Section 4.4 describes the Monte Carlo simulation procedure.

4.1 Lifecycle design

Households in our base specification are composed of a female (f) and a male (m) of equal age. The model periods are in months indexed by $t = 1, 2, \dots, T_{max}$, where T_{max} is the month of death of the last remaining survivor from the couple. Each member of the household is eligible to work and save starting from the first month of age 25 ($t = 1$). The retirement date is denoted T_{ret}, and our base case specifies an exogenous retirement age of 65, such that $T_{ret} = 480$. An individual may experience nonemployment during their potential working years, such that not all investors work the full 40 years. In the base case, we assume that individuals save $r_c = 10\%$ of their labor income for retirement, and no contributions occur during nonemployment periods. The assumed 10% contribution rate is close to the mean (11.7%) and median (11.0%) contribution rates for participants in Vanguard DC plans in 2023, including both employee and employer contributions [Vanguard (2024)].[17] We also assume that individuals earning less than $Y_{min} = \$15{,}000$ (in 2022 USD) in a given year forego contributing to their retirement plan, consistent with evidence of low retirement saving rates among this group [e.g., Vanguard (2024)].

At time $T_{ret}+1$, each individual leaves the workforce (ending either employment or nonemployment) and begins to draw from retirement savings and Social Security. We assume that investors withdraw $r_w = 4\%$ of their account balance at retirement in the first year and inflation-adjusted amounts calculated from this base withdrawal in subsequent years [i.e., the "4% rule" of Bengen (1994)]. In reality, retirees use a variety of withdrawal strategies. The 4% rule is ubiquitous in popular press and common retirement advice, so we use it as a simple heuristic for retirement withdrawals.[18] We also demonstrate that our main conclusions hold for alternative retirement withdrawal rules. We note that the outcomes of households who choose to annuitize fully at retirement will be reflected by our wealth at retirement results.

The Social Security Administration (SSA) reports conditional death probabilities at each age for females and males.[19] Our simulations incorporate gender-specific longevity risk, and the lifespan of each individual is randomly determined. Both the female and the male in each couple are alive at age 25, but one or both may die before retirement at age 65. There is considerable uncertainty over longevity outcomes. The 5th percentile of age at death for the couple (i.e., the last survivor) is 70.8 years, and the 95th percentile is 100.0 years. This uncertainty is an important feature to consider in

[17]Our assumed 10% contribution rate is also similar to Poterba, Rauh, Venti, and Wise's (2005, 2009) assumed 9% contribution rate to household retirement accounts.

[18]In Choi's (2022) review of the most popular personal finance books, he finds that seven of the 12 books offering explicit retirement spending advice recommend the 4% rule.

[19]See https://www.ssa.gov/oact/HistEst/PerLifeTables/2022/PerLifeTables2022.html.

assessing the ability of investment strategies to fund consumption through retirement (see the internet appendix for further details on the distribution of age at death).

Given the simulation design, the (unmodeled) consumption and potential survivor benefits from Social Security during the pre-retirement period are independent of the retirement investment strategy. As such, we do not study consumption in the pre-retirement period and do not include it in the utility calculations.

4.2 Household utility and portfolio choice problem

Household utility is determined by monthly retirement consumption and a bequest:

$$U(C,B) = \sum_{t=T_{ret}+1}^{T_{max}} \delta^t \frac{\left(C_t/\sqrt{H_t}\right)^{1-\gamma}}{1-\gamma} + \delta^{T_{max}} \theta \frac{(B+k)^{1-\gamma}}{1-\gamma}, \tag{1}$$

where C_t is real consumption in month t, H_t is the household's size, B is the real bequest, δ is the subjective discount factor, θ and k are bequest utility parameters, and γ is the coefficient of relative risk aversion. Following Duarte, Fonseca, Goodman, and Parker (2024), we scale household consumption by the square root of household size to reflect differences in consumption needs for couples versus singles and set $\delta = 1$ to equally weight the flow of utility during the retirement period. Our bequest utility specification follows De Nardi, French, and Jones (2010). We use their estimate for risk aversion of $\gamma = 3.84$, and we assume that this risk aversion coefficient applies to both consumption and bequest. De Nardi, French, and Jones (2010) estimate a bequest intensity of $\theta = 2{,}360$ when studying bequest utility alongside utility from annual consumption, and we multiply this parameter estimate by 12^{γ} to reflect the mechanical difference in scaling between monthly and annual consumption levels. Finally, we inflation-adjust their bequest curvature parameter k, which determines the extent to which bequests are viewed as luxury goods, and use $k = \$490{,}000$ in 2022 USD.[20]

The household has bounded rationality and uses the sparse max operator of Gabaix (2014). Under this framework, the household approaches its lifecycle problem in two steps. It first chooses how to allocate attention to the features of the problem that the household considers sufficiently important to affect portfolio choice significantly. The household then chooses portfolio weights to maximize expected utility within the simplified problem.

Household age is a particularly salient state variable for lifecycle portfolio choice. Age is impor-

[20]We do not consider utility from housing, which is an important asset for many households. Venti and Wise (1991) and Poterba, Venti, and Wise (2011) show that few households use reverse mortgages or otherwise decrease their home equity late in life.

tant for labor income profiles [e.g., Guvenen (2007)], proximity to retirement, and expected longevity. Survey evidence indicates the importance of age in household decisions, as Choi and Robertson (2020) find that "years left until retirement" is the most important self-reported factor for equity allocations. Investor age is also the sole household characteristic considered by the QDIA regulations that spurred the growth of status quo TDFs. As described in Section 2, investor horizon and age play central roles in the academic lifecycle literature. Seminal studies [e.g., Cocco, Gomes, and Maenhout (2005)] show that optimal portfolio weights vary substantially based on age. Based on this evidence, households will plausibly view age as more important for portfolio choice compared with other state variables like wealth, income, and past Social Security contributions.

If households view age as the most important state variable for asset allocation, then they will focus solely on age in Gabaix's (2014) framework given particular specifications for the attention function. In our base specification, the household chooses age-based portfolio weights across domestic stocks, international stocks, bonds, and bills to maximize expected utility:

$$\max_{\{\{w_t\}_{t=1}^{T_{max}}\}} \quad \mathbb{E}_0[U(C,B)] = \mathbb{E}_0\left[\sum_{t=T_{ret}+1}^{T_{max}} \frac{\left(C_t/\sqrt{H_t}\right)^{1-\gamma}}{1-\gamma} + \theta\frac{(B+k)^{1-\gamma}}{1-\gamma}\right], \tag{2}$$

$$\text{s.t.} \quad R_t^p = w_t' R_t, \tag{3}$$

$$\mathbb{1}' w_t = 1, \tag{4}$$

$$w_t \geq 0, \tag{5}$$

$$W_0 = 0, \tag{6}$$

$$W_{t+1} = \begin{cases} W_t(1+R_{t+1}^p) + S_{t+1} & \text{for } t \leq T_{ret}, \\ (W_t - D_{t+1})(1+R_{t+1}^p) & \text{for } t > T_{ret}, \end{cases} \tag{7}$$

$$D_{t+1} = \begin{cases} 0 & \text{for } t \leq T_{ret}, \\ \min\{\frac{1}{12}(r_w W_{T_{ret}}), W_t\} & \text{for } t > T_{ret}, \end{cases} \tag{8}$$

$$C_{t+1} = \max\{D_{t+1} + SS_{t+1}, SSI_{t+1}\} \quad \text{for } t > T_{ret}, \tag{9}$$

$$B = W_{T_{max}}, \tag{10}$$

where w_t is a 4×1 vector of portfolio weights in month t, R_t is a 4×1 vector of gross real returns on the four assets, R_t^p is the gross portfolio return, $\mathbb{1}$ is a 4×1 vector of ones, W_t is the household's end-of-month retirement wealth, S_t is the monthly flow of savings for the couple, D_t is the monthly retirement account withdrawal, SS_t is the couple's monthly combined Social Security benefit, and SSI_t

is their monthly Supplemental Security Income. Equations (4) and (5) represent the constraints faced by the couple on taking levered positions and short positions in risky assets. In subsequent analyses, we relax the restriction on portfolio leverage.

The optimal weights $\{w_t^*\}_{t=1}^{T_{max}}$ provide a sequence for age-based asset allocation. To mitigate the effects of simulation error on the estimated optimal weights, we restrict the weights to be equal across all months at a given age (e.g., the optimal age-35 weights satisfy $w_{121}^* = w_{122}^* = \cdots = w_{132}^*$). We also group ages 25-29 (when household savings are minimal) and ages 90 and above (when the sample of surviving households is small).

4.3 Lifecycle income

We model labor income using the model of Guvenen, Karahan, Ozkan, and Song (2021). Their flexible framework allows for investor heterogeneity, permanent and transitory income shocks, and employment and nonemployment states. They estimate the model to fit a large number of cross-sectional moments and time-series properties of lifecycle earnings data on millions of US workers from the SSA.

The annual income level for investor i ($i \in \{f, m\}$) at age $\tau + 24$ (for $\tau = 1, 2, \ldots, 40$) is given by

$$Y_\tau^i = (1 - \nu_\tau^i) e^{(g(\tau) + \alpha^i + \beta^i f(\tau) + z_\tau^i + \varepsilon_\tau^i)}, \tag{11}$$

where $g(\tau)$ is a quadratic polynomial that fits the well-known hump shape of lifecycle earnings, $f(\tau)$ is a linear function increasing in τ, α^i and β^i are investor-specific parameters that affect the expected level and slope of earnings, z_τ^i is a persistent earnings component following

$$z_\tau^i = \rho z_{\tau-1}^i + \eta_\tau^i, \tag{12}$$

and ε_τ^i is a transitory earnings shock. Both the permanent shocks (η_τ^i) and the transitory shocks (ε_τ^i) follow normal mixture distributions. Finally, $\nu_\tau^i = 0$ represents full-year employment, whereas $\nu_\tau^i = 1$ is full-year nonemployment. This nonemployment variable takes values as follows:

$$\nu_\tau^i = \begin{cases} 0 & \text{with prob. } 1 - p_\nu(\tau, z_\tau^i), \\ \min\{1, \phi_\tau\} & \text{with prob. } p_\nu(\tau, z_\tau^i), \end{cases} \tag{13}$$

where ϕ_τ follows an exponential distribution with mean $1/\lambda$, $p_\nu(\tau, z_\tau^i) = \frac{e^{\xi_\tau^i}}{1+e^{\xi_\tau^i}}$ is the nonemployment probability, and $\xi_\tau^i = a + bf(\tau) + cz_\tau^i + dz_\tau^i f(\tau)$ with $b < 0$, $c < 0$, and $d < 0$. As such, the probability of nonemployment is negatively influenced by the level of the persistent earnings component. This modeling feature generates persistence in the nonemployment state.

We assume annual income is divided evenly over months in the year. For months $12(\tau - 1) < t \leq 12\tau$, savings is given by

$$S_t^i = \begin{cases} \frac{1}{12}(r_c Y_\tau^i) & \text{for } Y_\tau^i \geq Y_{min}, \\ 0 & \text{for } Y_\tau^i < Y_{min}, \end{cases} \tag{14}$$

and

$$S_t = S_t^f + S_t^m. \tag{15}$$

The heterogeneity in earnings processes across investors is captured by two income parameters, α^i and β^i, and the initial state of the permanent income component, z_0^i. High (low) values for α^i and β^i designate investor types with high (low) levels and growth rates, respectively, for expected lifetime earnings, whereas high (low) z_0^i captures a tendency for high (low) early-career earnings. In our base case analyses, we set all three parameters for both members of the couple equal to their median values in the Guvenen, Karahan, Ozkan, and Song (2021) calibration, i.e., $(\alpha^i, \beta^i, z_0^i) = (0, 0, 0)$.

We simulate from the labor income model using the parameter estimates from the replication code of Guvenen, Karahan, Ozkan, and Song (2021) with the additional assumption that the income model applies equally to females and males. We scale the simulation output (which initially has no standard unit of measurement) to match the level of average log earnings in 2010 USD [Figure C.36 in Guvenen, Karahan, Ozkan, and Song (2021)] and then convert to 2022 USD by adjusting for the change in the consumer price index (CPI). Figure 2 plots the distribution of household income from our base case simulation as a function of household age. The mean reflects the well-known hump shape in earnings [e.g., Cocco, Gomes, and Maenhout (2005)], and the 10th and 90th percentiles imply considerable uncertainty in earnings.

Our simulations incorporate retirement income from Social Security benefits and the additional social safety net from Supplemental Security Income (SSI). Social Security benefits are calculated based on taxes paid on earnings during the working years. We use the formulas effective in 2022 to calculate Social Security benefits based on each worker's earnings. We incorporate spousal and survivor benefits in the retirement period. In the internet appendix, we provide full details of the Social Security benefit calculations. Finally, SSI is available to retirees with little other income. The maximum monthly benefit in 2022 is $1,261 for couples and $841 for singles.

4.4 Simulation procedure

Our optimization approach finds approximately optimal portfolio weights using Monte Carlo simulation. For a given portfolio strategy, we simulate lifecycle outcomes and calculate household utility following equations (2) to (10). We use grid search to find the portfolio weights that maximize the average lifetime household utility across simulations.

We describe the simulation steps for a candidate sequence of portfolio weights, $\{w_t\}_{t=1}^{T_{max}}$. We simulate 1,000,000 lifecycle outcomes, with the following steps in each draw:

1. We determine the lifespan of the household. We generate random longevity using conditional mortality probabilities, and we assume that the probability of death is equal across the 12 months at a given age. In the base case, we denote the realized lifespans of the female and the male as T_f and T_m and the couple's lifespan as $T_{max} = \max\{T_f, T_m\}$.

2. We adopt a stationary block bootstrap approach in the spirit of Politis and Romano (1994) to draw a time series of monthly real returns for the four asset classes that covers the couple's lifespan. We randomly select a starting country-month observation in the full sample and draw a block of consecutive returns from the same country to capture time-series dependencies in asset returns. The block length is drawn from a geometric distribution. The average block length is 120 months in the base case, so the blocks reflect long-term time-series properties of returns. A set of all four asset class returns is drawn from each selected country-month to preserve cross-sectional dependencies across assets, and we denote a monthly real return vector as

$$R_t = [R_t^{Domestic\ stocks} \quad R_t^{International\ stocks} \quad R_t^{Bonds} \quad R_t^{Bills}]'. \tag{16}$$

   In the event that the asset return data from a country-period is insufficient to fill a return block (i.e., the end of the sample for that country occurs before the number of return vectors in the block equals the drawn block length), we draw a random country and continue to fill the return block with return data from the beginning of that country's sample (i.e., we use a stationary block bootstrap approach to avoid undersampling). We repeatedly draw blocks of returns from random countries and periods until we produce a time series of asset class returns that spans the couple's lifetime. The final bootstrap draw of asset class returns in the iteration is $R = \{R_1, R_2, \ldots, R_{T_{max}}\}$.

3. Given the investment strategy $\{w_t\}_{t=1}^{T_{max}}$, we compute a monthly time series of portfolio returns over the couple's lifetime. The portfolio return in month t is $R_t^p = w_t' R_t$.

4. The couple begins with no wealth in savings, $W_0 = 0$. We calculate the evolution of wealth during the working years, considering savings from labor income and returns on invested wealth as previously described.

5. At retirement, the couple stops working and saving. We calculate the evolution of wealth during the retirement years, considering withdrawals and returns on invested wealth as previously described. If the household's wealth is depleted at any time during retirement, it remains at zero until death. The household also receives monthly Social Security benefits. The couple is supported by SSI if their income falls below the threshold. The couple's bequest is all remaining wealth at the death of the last surviving spouse.

6. The household's time series of monthly consumption during retirement and its bequest determine utility according to equation (1).

The average household utility across the 1,000,000 draws is the Monte Carlo estimate of the expected utility from the investment strategy $\{w_t\}_{t=1}^{T_{max}}$. For each strategy, we compile statistics for variables of interest, such as wealth at retirement and retirement-period drawdown.[21]

We use this simulation approach to estimate expected utility for candidate portfolio strategies and several benchmark strategies. The optimization procedure for the base case of age-based weights requires an iterative approach, as the optimal weights at any given age depend on the weights chosen for all other ages. In each iteration, we use a grid search to optimize the weights at each age conditional on the previous iteration's optimal weights for other ages. We iterate until both the weight changes across successive iterations become consistently small and the economic differences become negligible.

We make utility comparisons across strategies by running simulations as described above but with a range of potential savings rates to compute an equivalent savings rate. Given the expected retirement utility from a current investment strategy (e.g., the optimal age-based allocation) with the 10% base savings rate during the working years, we find the savings rate associated with an alternative investment strategy that provides the same expected retirement utility.

[21] Maximum portfolio drawdowns are calculated as the largest real negative cumulative return relative to the previous peak. The working-period drawdown occurs entirely within the working years. The retirement-period drawdown begins with a peak that could occur during either the working years or the retirement years. That is, falling asset prices in the late working years can contribute to our measured retirement drawdown.

5 Results

In this section, we examine optimal portfolio choice in our lifecycle model. We present the optimal age-based strategy and the corresponding utility gains relative to four benchmark strategies (Section 5.1) and then compare strategy performance in the pre-retirement and post-retirement periods (Section 5.2). We also demonstrate the importance of our bootstrap sampling approach and the specification of the investment opportunity set for determining the optimal age-based policy (Section 5.3). We then use a fixed-weight specification to evaluate the sensitivity of the optimal investment policy to a wide range of design assumptions (Section 5.4).

5.1 Optimal age-based strategy

Panel A of Table III summarizes the optimal age-based strategy, and Figure 3 illustrates the sequence of portfolio weights from age 25 to 95. A household with longevity of 95 years allocates 99% to equity on average, with an average split of 31% domestic stocks and 69% international stocks. We observe a slight tilt toward higher domestic allocations during working years relative to retirement years. The household remains fully invested in stocks except for a brief period early in retirement. Upon retirement at age 65, the optimal allocation shifts to 26% domestic stocks, 47% international stocks, 0% bonds, and 27% bills. By age 68, however, the equity allocation again exceeds 90%.

Our key finding is that the optimal age-based policy eschews fixed income investments in favor of an all-equity portfolio for nearly all of the lifecycle. This result may appear counterintuitive, given the vaunted diversification benefits and safety offered by bonds. As discussed in the context of Table I, however, bonds become riskier and more correlated with domestic stocks as the investment horizon lengthens. The optimal allocation instead reflects the superior diversification benefits and growth potential offered by international stocks.

The modest allocation to bills early in retirement represents the household's response to the sequence-of-returns risk created by the 4% rule's fixed real withdrawal structure. To illustrate, Figure 4 shows the optimal age-based strategy under an alternative withdrawal rule—one in which the household withdraws 4% of current retirement wealth rather than 4% of wealth at the retirement date. Given this flexible withdrawal structure, there is no sequence-of-returns risk and the household refrains from investing in fixed income at all ages. The base case allocation in Figure 3 reflects tactical cash reserves due to the rigidity of the 4% rule rather than the attractiveness of bills.

Holding cash reserves in bills also provides little economic value. Consider a household that maintains an all-equity strategy with the same domestic-international split as the optimal age-based strategy. To gain the same expected utility over retirement consumption and bequest as a 10.00% savings rate in the optimal age-based strategy, the all-equity household would save 10.05%. Little is lost by investing exclusively in equity.[22]

Panel B of Table III shows asset weights for four benchmark strategies and provides an analysis of the economic differences compared with the optimal age-based strategy. The benchmark strategies are a 100% allocation to bills, a 100% allocation to domestic stocks, a balanced strategy with a 60% allocation to domestic stocks and a 40% allocation to bonds, and the TDF shown in Figure 1. The bills strategy was a common default in the pre-PPA era, and the balanced and TDF strategies are QDIAs under the PPA. The table reports the savings rates that provide households with the same expected utility as a 10.00% savings rate in the optimal age-based strategy.

The optimal age-based strategy provides higher expected utility than the benchmarks by construction, but the equivalent savings rates reveal large economic magnitudes of utility differences. The three fixed-weight benchmarks perform poorly. To achieve the same degree of wellbeing as under the optimal age-based policy, the couple must save 56.71% (bills), 16.42% (domestic stocks), or 19.44% (balanced) of its annual income during the working years. The status quo TDF adopts an age-based strategy in an attempt to optimize over the lifecycle, but we find a couple must save 16.27% (i.e., 63% more) to achieve the same expected utility with the TDF as with the optimal age-based strategy. Overall, Table III implies that a household currently heeding conventional investment advice can achieve a large increase in utility by reallocating to an internationally diversified equity portfolio.[23]

5.2 Lifecycle investment strategy performance

The previous section demonstrates that the optimal weights are approximately one-third domestic stocks and two-thirds international stocks at all ages, with little fixed income exposure. In this section, we examine simulation results for couples who adopt this optimal investment strategy or one of the alternatives from Panel B of Table III. For each strategy, we study four retirement saving outcomes: (i)

[22]We also consider a boundedly rational household that conditions on wealth (i.e., above- or below-median wealth) in addition to age. The optimal weights are qualitatively similar to the optimal age-based strategy, and the equivalent savings rate is 9.96%.

[23]The magnitude of the estimated welfare gains depends on household risk aversion, but our conclusion that the optimal strategy provides economically large gains relative to the TDF holds for all γ values between zero and ten. As is tradition in the literature [e.g., Campbell, Cocco, Gomes, and Maenhout (2001)], we take a partial equilibrium view of lifecycle investing and do not consider equilibrium effects of investor shifts across investment strategies. As Heaton and Lucas (2000) state, "Although one could look at portfolio choice in a general equilibrium framework, unless the model were capable of generating a realistic returns process the results on portfolio choice would also be suspect."

the distribution of wealth at retirement, (ii) the distribution of the income replacement rate to describe the consumption stream in retirement, (iii) the probability of exhausting financial wealth prior to death, and (iv) the distribution of wealth at death. We also consider two intermediate outcomes: (i) the distribution of the maximum drawdown during the working years and (ii) the distribution of the maximum drawdown during the retirement years.

5.2.1 Pre-retirement period

Panel A of Table IV summarizes the distributions of retirement wealth. For each strategy, the table shows the mean, standard deviation, and percentiles of the wealth distribution at the beginning of the retirement period across 1,000,000 bootstrap simulations. Panel A of Figure 5 provides a visual summary of the five strategies using box-and-whiskers plots. The middle line is the median, the box designates the interquartile range, and the whiskers extend to the 10th and 90th percentiles. The exhibits report wealth at retirement in millions of 2022 USD.

For context in interpreting the wealth levels, couples save $0.24 million on average. As reported in Table C.I, about 2.3% of households have both members die prior to retirement age. For these couples, bequests occur prior to retirement and wealth at retirement is $0.

The optimal strategy, which is fully invested in stocks during the working years, outperforms the alternatives in generating wealth at retirement. Bills perform poorly in wealth accumulation, with an average retirement wealth balance of only $0.27 million. The poor performance demonstrates that money market and stable value funds are ineffective retirement saving tools when used in isolation and provides support for the changes to default DC plan options brought about by the PPA. Investing in domestic stocks produces $1.02 million in wealth on average (annual withdrawal of $40,800 given our 4% retirement withdrawal rule), which is higher than the averages for the two QDIAs of $0.71 million for the balanced portfolio (withdrawal of $28,400) and $0.77 million for the TDF (withdrawal of $30,700). The optimal age-based strategy generates the most wealth on average at $1.06 million, which supports an annual real withdrawal of $42,500.

More important, the percentiles in Panel A of Table IV show that the distribution of wealth for the optimal strategy is preferable to the distributions for all other strategies. Concentrating on the poor outcomes, the optimal strategy has a 5th percentile of $0.13 million versus $0.03 million for bills, $0.07 million for domestic stocks, $0.06 million for the balanced strategy, and $0.09 million for the TDF. The optimal strategy provides households with impressive upside, and its international diversification limits downside risk during the saving years.

5.2.2 Retirement period and bequest

Panel B of Table IV and Panel B of Figure 5 summarize the distribution of the income replacement rate during retirement for each strategy. The reported replacement rates are calculated as the mean of monthly household retirement consumption divided by the mean of monthly household income during the working ages of 25 to 64.[24] The table reports that the two QDIA strategies (along with Social Security benefits) allow couples to achieve full income replacement, on average. The mean replacement rate is 1.03 for the TDF, for example, although 62% of couples have a replacement rate below one. The optimal strategy performs best by generating a mean replacement rate of 1.24, and more than half (56%) of couples achieve full replacement or better. The left-tail outcomes for the optimal portfolio also exceed those for the QDIA strategies.

Preserving wealth in retirement is crucial, and retirees who exhaust their savings must rely solely on their Social Security and SSI benefits. Panel C of Figure 5 plots the probability of financial ruin, defined as reaching $0 in wealth prior to the death of the last survivor in the household. Couples investing in "safe" bills and using the 4% rule have a 38.9% chance of running out of savings. Fully investing in domestic stocks is better with a 17.1% ruin probability, but this risk is likely higher than most households would like to face. The two QDIAs, which invest in fixed income to preserve wealth, also fail to generate reliable streams of retirement income. The balanced strategy has a ruin probability of 16.9%, and the status quo TDF has an even higher probability of 19.7%. Recall from Figure 1 that the TDF invests just 17% in equity throughout much of retirement. The large allocations to bonds and bills do little to prevent poor retirement outcomes. The ruin probability for the optimal age-based strategy is low in comparison at 6.7%. Equity offers strong potential for additional investment gains during retirement, and international diversification is crucial for capital preservation.

Panel C of Table IV and Panel D of Figure 5 display distributional statistics for real wealth upon the death of the last survivor in the household. A couples's bequest is $0 if they experience financial ruin prior to death. The optimal strategy substantially outperforms the other strategies in providing bequests. The mean wealth at death is $2.66 million for the optimal strategy versus $0.08 million for bills, $2.61 million for domestic stocks, $1.10 million for the balanced strategy, and $0.72 million for the TDF. The means are heavily affected by right-tail outcomes. The median couple generates a bequest of $0.96 million with the optimal strategy, which is far larger than for bills ($0.02 million), domestic stocks ($0.59 million), the balanced strategy ($0.35 million), or the TDF ($0.27 million). The large bequests for the optimal strategy reflect both the tendency for stocks to help in building wealth and the

[24] The replacement rate is equal to 0.00 for couples who are deceased prior to retirement.

relative safety of the strategy in preserving wealth during retirement.[25]

5.2.3 Portfolio drawdowns

The focus on the four retirement outcomes summarized in Panels A to D of Figure 5 glosses over important intermediate portfolio performance measures. Investors and regulators may focus on short-term losses as a metric for the suitability of retirement strategies. To study this issue, Table V and Panels E and F of Figure 5 report the largest portfolio drawdowns during the working years and retirement years for each strategy. The reported drawdowns are the largest peak-to-trough declines in real asset values for a given strategy, and they are expressed in decimal form in the table.

During the working years, each strategy produces large real drawdowns on average. Panel A of Table V shows average maximal drawdowns of 42% for bills, 67% for domestic stocks, 54% for the balanced strategy, 52% for the TDF, and 55% for the optimal strategy. The optimal strategy's 55% average drawdown would cause discomfort for even the most stalwart investors, but each strategy that attempts to provide long-term appreciation is subject to similarly large average intermediate losses. Investors, advisors, and regulators are likely most concerned about the largest potential drawdowns, and the optimal strategy outperforms the alternatives in the right tail of the drawdown distribution. The 95th percentile drawdown of 77% for the optimal strategy is favorable relative to drawdowns of 96% (bills), 96% (domestic stocks), 92% (balanced), and 81% (TDF).

Retirement-period drawdowns are likely an even larger concern for households. Panel B of Table V reports average maximal drawdowns during retirement of 46% for bills, 62% for domestic stocks, 50% for the balanced portfolio, 40% for the TDF, and 47% for the optimal strategy. According to this metric, the TDF performs better than the optimal portfolio. Based on avoiding the largest drawdowns, however, the optimal strategy is the best. The 95th percentile for the optimal portfolio is a 73% drawdown versus an 89% drawdown for the TDF.

We note that the superior performance of the optimal age-based strategy in the four outcomes (generating wealth at retirement, producing retirement income, avoiding financial ruin, and providing a bequest) occurs despite the potential for relatively large intermediate drawdowns. Experiencing large drawdowns is painful for investors, and many investors will have a natural inclination to abandon their strategies at inopportune times when drawdowns occur. An important policy issue is how to manage

[25] In the internet appendix, we further study household retirement outcomes. We show that the favorable performance of the optimal age-based strategy relative to the benchmarks in achieving retirement outcomes (i) is partially attributable to continued wealth generation in retirement to combat longevity risk, (ii) persists even if the domestic stock market has a poor realization during retirement, (iii) is strongest if realized inflation in retirement is high, and (iv) occurs even if the realized correlation between domestic and international stock markets is high.

this behavior, as the optimal strategy relies on maintaining high equity allocations through good times and bad. An additional policy issue is the extent to which the average drawdown is emphasized relative to the worst potential drawdowns. The TDF provides lower average and median retirement drawdowns than the optimal age-based approach, but it has poor worst-case scenarios when high inflation erodes the real value of bonds in retirement.

5.3 Bootstrap design and investment opportunity set

Our findings on the strong performance of the optimal age-based strategy—which is virtually all equity throughout the lifecycle—may be surprising given conventional wisdom. Our lifecycle model detailed in Section 4 has two important differences compared with common approaches: (i) we adopt a block bootstrap with long blocks (120 months on average) to preserve time-series dependencies in returns and (ii) we include international stocks in the investment opportunity set. In Figure 6, we study the impact of these two design choices. In each specification, we use either a block bootstrap or an IID bootstrap (i.e., one-month blocks), and we include either domestic assets or all assets (i.e., including international stocks). The figure considers the four combinations of these two choices, with the all-block approach in Panel D being repeated from Figure 3 (i.e., the base case).

Figure 6 reveals important differences in optimal age-based strategies depending on design choices. In all cases, pre-retirement portfolios are fully invested in stocks. At age 65, however, the strategies differ sharply. The domestic-IID case in Panel A has 14% in bonds and 48% in bills versus 0% in bonds and 27% in bills for the all-block case in Panel D. Each design features rising optimal equity allocations during the retirement years, but stock-bond allocations differ substantially across the specifications. Across the four cases, we observe two patterns with respect to design choices. First, using a block bootstrap reduces the attractiveness of bonds relative to using an IID bootstrap. This pattern reflects the poor long-term bond return properties evident in Table I. Second, adding international stocks to the investment opportunity set reduces the allocation to bonds and bills. Households invest in fixed income at every age post-retirement when limited to domestic assets, but do not with access to international stocks. Consistent with Table I, international stocks are quite attractive relative to bonds as a diversification tool, especially at long horizons.

Figure 6 helps to reconcile our findings of strong performance for the optimal age-based strategy, which nearly exclusively invests in equity, with the conventional advice that retirees should invest in "conservative" bonds and bills. A domestic-IID assumption is made in many academic and practitioner outlets (e.g., those using monthly return moments from US stocks and bonds to calibrate models). This similarity may explain the widespread support for age-based, stock-bond investing. But the domestic-IID method misses two important aspects of modeling the investment opportunity set. First, returns are not IID. Given that the time-series dependencies in returns have an important impact on optimal asset allocation, it seems difficult to justify an IID assumption. Second, international stocks are an attractive asset class. They provide diversification to domestic stock investors and also offer high expected returns. Investing in equity throughout one's lifetime is supported by the data when we account for these features of the investment opportunity set.

5.4 Optimal fixed-weight strategy and lifecycle modifications

A pronounced feature of the optimal age-based strategy summarized in Figure 3 is the stability of asset class weights over the lifecycle. This pattern suggests that a simpler, fixed-weight strategy may yield economically similar outcomes. In this section, we consider a bounded rationality household that approaches a simplified version of the utility maximization problem in Section 4.2, such that the couple does not condition weights on age. This household maximizes expected retirement utility by choosing a fixed-weight allocation throughout the lifecycle (i.e., $w_t = w$ for $t = 1, 2, \ldots, T_{max}$). We use the simulation procedure in Section 4.4 with a grid search over potential allocations to domestic stocks, international stocks, bonds, and bills to find the optimal fixed-weight strategy.

The optimal fixed-weight strategy achieves virtually identical economic outcomes to those of the more complex, age-conditioned strategy. Under the optimal fixed-weight strategy, the couple invests 33% in domestic stocks, 67% in international stocks, 0% in bonds, and 0% in bills at all ages. These fixed weights closely align with the average time-series weights from the optimal age-based policy. To achieve the same expected retirement utility as a 10.00% savings rate in the optimal age-based policy, a couple with the optimal fixed-weight policy saves 10.07%. The internet appendix demonstrates that the optimal age-based and fixed-weight strategies provide similar performance in the pre-retirement and post-retirement periods according to the metrics in Section 5.2. The two strategies are nearly equivalent in wealth accumulation. The tactical cash reserves in the age-based strategy lead to slight improvements relative to the fixed-weight strategy for ruin probability (6.7% versus 7.0%) and average retirement-period drawdown (47% versus 48%), at the cost of a lower average bequest ($2.66 million versus $2.94 million).

Given the similarity in economic outcomes across the optimal age-based and fixed-weight strategies, the fixed-weight design provides a convenient setting for quantifying welfare losses from deviating from the optimal allocation. As shown in the internet appendix, expected utility as a function of the allocation to domestic stocks is relatively flat around the optimum of 33% domestic stocks and 67% international stocks. All allocations ranging from 11% domestic and 89% international to 55% domestic and 45% international have equivalent savings rates below 10.50% (relative to the optimal fixed-weight strategy). This finding gives real-world investors considerable latitude in choosing equity strategies. For example, a US investor may feel comfortable investing over half of their wealth in the domestic market given the US's large global weight. As long as investors avoid overly large domestic equity allocations, the utility costs are small even if they deviate from the optimum but remain invested in stocks. The welfare losses incurred by deviating from the optimal portfolio by adding fixed income, in contrast, are substantially greater.

In the remainder of this section, we assess the sensitivity of our main conclusions to a broad set of modifications to the household's investment problem. Given the scope of the modifications considered, the computational demands of the iterative approach to solving for optimal age-based allocations, and the close alignment in economic outcomes between the age-based and fixed-weight strategies, we focus exclusively on fixed-weight specifications for these sensitivity analyses.

5.4.1 Age- and income-based contribution rates

Our base case specifies a constant retirement contribution rate. We consider contribution rates that vary based on age and income. We use the realized contribution rates estimated by Parker, Schoar, Cole, and Simester (2023) using account-level data on US retirement savers. They divide savers into terciles based on income and examine investor contribution rates by age groups (i.e., ages 25-27, ages 28-30, and so on).[26] We place each retirement saver in our simulation into one of the age- and income-based groups based on the age and income cutoffs reported by Parker, Schoar, Cole, and Simester (2023), such that each individual in our simulation has a time-varying contribution rate that matches the behavior of US investors. This modification does not affect the optimal fixed-weight strategy. The couple chooses an allocation of 33% in domestic stocks, 67% in international stocks, 0% in bonds, and 0% in bills.

5.4.2 Optimal retirement timing

We next examine optimal retirement timing. We allow households to optimize across portfolio choice and retirement age, with potential ages ranging from 62 to 70 (the earliest and latest ages to claim Social Security). We assume that the couple claims Social Security upon retirement. The investors

[26]Fagereng, Holm, Moll, and Natvik (2021) show that the net contribution rate is relatively constant as a function of wealth, so we concentrate on the age- and income-based savings rates in Parker, Schoar, Cole, and Simester (2023).

optimize utility over a bequest and consumption from age 62 until death, and their consumption prior to retirement (i.e., from age 62 through the chosen retirement age) is set to their earnings minus their retirement contributions.

To find the optimal retirement ages, we first classify couples into terciles across each of three dimensions as they are turning 62: (i) current earnings, (ii) current retirement wealth, and (iii) expected Social Security benefit level. This procedure classifies each couple into one of 27 groups. We find the optimal retirement age for each group. As detailed in the internet appendix, the optimal retirement ages range from 62 to 70. The optimal age is increasing in current income, decreasing in current wealth, and decreasing in Social Security benefits. We find the fixed-weight asset allocation strategy that is jointly optimal with the retirement ages. Incorporating optimal retirement timing produces an optimal asset allocation of 33% in domestic stocks, 67% in international stocks, and 0% in bonds and bills.

5.4.3 Time-varying investment allocations conditional on the market state

We study whether the couples in our simulations prefer to adopt time-varying allocations that depend on the market state. The price-dividend ratio of the domestic stock market is a prominent state variable in the asset allocation literature, so we allow our investors to consider this variable. We augment our dataset of asset class returns with a lagged price-dividend ratio for each country-month before running the bootstrap. As such, entering each month, the couple is aware of the current valuation level of domestic stocks. We divide the market states into quintiles, creating groups of country-months that range from low price-dividend ratios (from 0.00 to 18.76) to high price-dividend ratios (from 43.67 to infinity). We allow the couples to choose a different asset allocation in each quintile, and we find the jointly optimal set of state-dependent weights.

Table VI shows the optimal conditional allocations. Panel A repeats allocations for the optimal fixed-weight strategy, which does not condition on the market state. Panel B shows the optimal weights for each price-dividend quintile. When the domestic price-dividend ratio is low, investors weight domestic stocks heavily at 65% and allocate the remaining 35% to international stocks. In the middle three quintiles, the allocations are similar to the unconditional optimal strategy. In the quintile with the highest price-dividend ratios, the couples reduce the domestic stock allocation to 16%, increase international stock exposure to 75%, and invest 9% in bonds. Thus, when domestic stock prices are very high, the couples optimally allocate a small portion of their wealth to bonds.

To measure the economic gains from considering the market state relative to an unconditional fixed-weight strategy, we calculate the equivalent savings rate. To achieve the same expected utility as the couple saving 10.00% with the optimal fixed-weight strategy, a couple using the conditional strategy saves 9.73%. In untabulated results, we find that nearly 80% of the economic gain from conditioning is attributable to varying the domestic-international stock allocation rather than to including bonds.[27]

5.4.4 Additional lifecycle modifications

Table VII presents optimal fixed-weight allocations under alternative lifecycle designs. For each specification, we report the optimal asset class weights. The last column of the table shows the amount of borrowing as a percentage of wealth for the specifications in which leverage is allowed. For reference, Panel A of Table VII reports the optimal fixed-weight policy, which is fully invested in equities with 33% allocated to domestic stocks and 67% to international stocks.

Panels B through J of Table VII show that our results are insensitive to modifying several aspects of the simulation design:

- the underlying sample of asset class returns restricted to the post-World War II period (i.e., October 1945 to December 2023),
- the underlying sample restricted to countries with relatively large populations (i.e., a given country's data are included in the sample only for the period after the country's population first reaches 0.5% of the world population),
- the underlying sample restricted to countries with relatively developed equity markets (i.e., a given country's data are included in the sample only for the period after the country's market capitalization-to-GDP ratio first reaches 0.5),
- the underlying sample to exclude all United States data,
- the average block length in the stationary block bootstrap to be 60 months or 240 months,
- the risk aversion parameter from $\gamma = 0.5$ to $\gamma = 10.0$,[28]
- the weight on bequest from $\theta = 0$ (i.e., no bequest motive) to $\theta = \infty$ (i.e., utility only from bequest),

[27]If we restrict investors from holding bonds and bills, the optimal conditional strategy invests 64%, 27%, 30%, 30%, and 22% in domestic stocks across the five states with the remainder in international stocks. The equivalent savings rate (relative to the optimal fixed-weight strategy) for this all-equity conditional strategy is 9.79% compared with 9.73% for the conditional strategy in Table VI.

[28]Calvet, Campbell, Gomes, and Sodini (2025) estimate risk aversion in the cross section of Swedish households. The risk aversion parameter of 7.5 is close to their mean of 7.57, and their standard deviation of 1.06 implies that the parameters of 5.0 and 10.0 are roughly two standard deviations from the mean. Therefore, the large majority of households lie in this range according to Calvet, Campbell, Gomes, and Sodini's (2025) estimates.

- the household utility specification to scale consumption by H_t (i.e., household size) instead of $\sqrt{H_t}$,
- the household utility specification to include a subjective discount factor that we set to $\delta = 0.98^{1/12}$ (i.e., an annualized discount factor of 0.98),
- the withdrawal strategy to be a constant, real withdrawal rate of $r_w = 3\%$ or $r_w = 5\%$,
- the withdrawal strategy to withdraw an annualized 4% of wealth at the beginning of each month,
- the retirement age to be 62, 67, or 70 (i.e., T_{ret} to equal 444, 504, or 540),
- the contribution rate to be $r_c = 5\%$ or $r_c = 15\%$,
- the lower income limit for retirement saving to be $Y_{min} = \$0$ or $Y_{min} = \$45{,}000$, and
- the household type to be a single female, a single male, a female couple, or a male couple.

None of these alterations has a material impact on the optimal fixed-weight strategy. The weights in bonds and bills are 0% in every case, and the weight in domestic stocks varies between 27% and 45%.

Investor types

Our base analysis studies households with median parameter values in the lifetime earnings model of Guvenen, Karahan, Ozkan, and Song (2021). Panel K of Table VII studies four household types differing along two dimensions: low or high human capital $[(\alpha^i, \beta^i)]$ and low or high initial income $[z_0^i]$. These parameter combinations produce a variety of lifetime earnings profiles (e.g., saving concentrated earlier versus later in the working years) and total lifetime earnings levels. Despite the differences in savings behavior across households and the relative importance of savings versus Social Security, the optimal strategies are virtually identical. The results suggest a one-size-fits-all approach to lifetime portfolio choice.

Correlated labor income and stock returns

Panel L of Table VII explores the effect of correlation between the persistent earnings shock in Guvenen, Karahan, Ozkan, and Song's (2021) model and domestic stock returns. The earnings shocks are annual, so we relate them to annual stock returns. We first generate a distribution of compounded annual stock returns from our block bootstrap approach. We then use a Gaussian copula approach to introduce correlation between the bootstrap distribution of annual stock returns and the normal mixture distribution of the earnings shock. The base case has 0.0 correlation, and Panel L considers correlations ranging from 0.1 to 0.5.[29]

Introducing positive correlation between income and domestic stock returns decreases the optimal weight in domestic stocks. At a modest correlation of 0.1, the optimal strategy is 30% in domestic stocks and 70% in international stocks. At a high correlation of 0.5, the weight in domestic stocks is down to 18% with the remaining 82% allocated to international stocks. None of the couples switches to bonds when shifting their portfolios away from domestic stocks.[30]

Leverage

Our primary fixed-weight case considers couples who are constrained from taking on leverage in their retirement savings vehicles. This constraint is realistic for most investors and can affect the optimal portfolio allocation [e.g., Frazzini and Pedersen (2014)]. Asness (1996) argues, however, that investors should use leverage to invest in a balanced portfolio with 60% domestic stocks and 40% bonds rather than invest 100% in equities with no leverage.[31] We relax the constraint on leverage to study hypothetical investors who are able to borrow in their retirement savings accounts.

We model leverage as follows. We assume that investors will pay the prevailing yield on local government bills plus a spread. We consider (i) a high spread of 6.50% that is estimated by Davis, Kubler, and Willen (2006) from household borrowing costs and near the median broker margin rates as of April 2024, (ii) a medium spread of 1.40% that equals the lowest broker margin rate as of April 2024, and (iii) a low spread of 0.37% estimated by van Binsbergen, Diamond, and Grotteria (2022) as an average risk-free rate spread over government bills using derivative prices. We consider borrowing levels between 0% and 100% of wealth in increments of 5%. The 100% upper limit is in line with US regulations on margin for stocks.

The couples who have access to leverage jointly optimize over asset class weights and leverage levels. Panel M of Table VII reports that a couple subject to the high borrowing spread does not wish to use

[29] The literature diverges on the magnitude of the correlation between income and stock returns. Fama and Schwert (1977) and Cocco, Gomes, and Maenhout (2005), among others, find little relation. Davis and Willen (2000) and Campbell, Cocco, Gomes, and Maenhout (2001) estimate heterogeneous correlations with respect to educational attainment, with highly educated individuals having correlations as high as 0.3 to 0.5. Benzoni, Collin-Dufresne, and Goldstein (2007) model cointegration between dividends and labor income, and their model implies a correlation around 0.5 between stock returns and returns to human capital throughout most of the working years. We consider a wide range of correlations that spans these estimates.

[30] We also study the extreme case of perfect correlation considered by Bodie, Merton, and Samuelson (1992) and find that the optimal fixed-weight allocation is 3% domestic stocks, 97% international stocks, 0% bonds, and 0% bills. This result is consistent with the pattern in Panel L of Table VII.

[31] We compare Asness's (1996) preferred strategy of 60% domestic stocks, 40% bonds, and 55% borrowing with the optimal 100% equity strategy from our fixed-weight specification with no leverage. His strategy has an equivalent savings rate of 19.14%, indicating that levering a balanced portfolio leads to a large utility loss relative to the optimal fixed-weight, all-equity strategy with no leverage.

leverage. As such, the no-leverage constraint in our primary fixed-weight specification is not binding for this household. At the medium spread, the couple optimally borrows 55% of their wealth. Rather than levering up a balanced portfolio, as advocated by Asness (1996), the couple continues to choose an all-equity portfolio with 34% in domestic stocks and 66% in international stocks. These results show that households who pay realistic margin rates optimally choose all-equity strategies.

Panel M of Table VII shows that bonds enter the optimal allocation with the lowest spread of 0.37%. This borrowing spread from van Binsbergen, Diamond, and Grotteria (2022) reflects the risk-free rates implicit in futures and other derivatives. Few households currently trade derivatives in their retirement accounts. Leveraged strategies must be actively managed to maintain desired leverage, and leveraged products are often seen as unsuitable for long-term investors as a result of performance drags from frequent re-leveraging [e.g., Cheng and Madhavan (2009)]. To the extent that households invest through institutions to achieve this low borrowing cost, management fees would add to the cost of using leverage.[32] We study hypothetical investors who can access the risk-free borrowing rates without paying a management fee or transaction costs. With risk-free borrowing rates, optimal borrowing hits the cap at 100% of wealth. The optimal portfolio allocations are 28% in domestic stocks, 57% in international stocks, 15% in bonds, and 0% in bills. Thus, the couples remain heavily invested in stocks, but they have a small allocation to bonds. In the internet appendix, we demonstrate that the main conclusions from Panel M of Table VII hold for household risk aversion coefficients between 0.5 and 10.0.

5.4.5 American exceptionalism

As described in Section 3, we model forward-looking returns using a broad set of developed countries. Some contend that the US stock market is "special," however, such that restricting to the relatively short sample of US data could be sensible for Americans. One view of American stock market exceptionalism is that high historical returns reflect investors' past learning about US structural advantages, and current investor beliefs manifest as high valuation ratios and lower future expected returns [e.g., van Binsbergen, Hua, Peeters, and Wachter (2025)]. Reliance on the US sample requires a different view—common among practitioners—that the US will continue to outperform foreign markets and generate high future returns.[33]

We explore the role of American exceptionalism beliefs on optimal portfolio choice. We mix the

[32]Expense ratios for leveraged ETFs vary. The largest leveraged ETF, ProShares UltraPro QQQ, currently charges an annual expense ratio of 0.84%. The average leveraged ETF expense ratio is 1.04% (https://www.etf.com/topics/leveraged).

[33]See, for example, https://www.blackrock.com/us/financial-professionals/insights/reasons-to-remain-overweight and https://www.bloomberg.com/opinion/articles/2025-01-13/us-exceptionalism-doubting-america-can-still-cost-you-a-lot-of-money.

return distributions implied by the full developed country sample and the US sample with weights that reflect an investor's belief that the US has more favorable return characteristics. To implement this approach, we run the simulation with $(100-x)\% \times 1{,}000{,}000$ iterations using the developed sample and $x\% \times 1{,}000{,}000$ using the US sample, where $x\%$ is an investor's subjective probability that the US is special. The optimal weights maximize expected utility conditional on the belief.

Our analysis reveals that optimal allocations are sensitive to beliefs. A couple with complete conviction in American exceptionalism opts for a fixed-weight allocation of 100% in US stocks. Even a small degree of doubt introduces international diversification—a 90% exceptionalism belief corresponds to an optimal fixed-weight portfolio of 96% US stocks and 4% international stocks. A 50% belief that future US returns will be special (i.e., that prices do not yet reflect US structural advantages) leads to an optimal portfolio of 60% US stocks and 40% international stocks, closely aligning with current global market weights. Thus, the common practice of overweighting US stocks [e.g., French and Poterba (1991)] requires strong conviction in continued US stock market outperformance. We provide results for the full spectrum of exceptionalism beliefs in the internet appendix.

6 Conclusion

We approach the household's lifecycle investing problem with a particular emphasis on modeling the investment opportunity set. We employ a nonparametric block bootstrap method that preserves time-series and cross-sectional dependencies in asset returns and avoids distributional and parametric assumptions about the underlying data generating process. The household optimally chooses an internationally diversified equity portfolio, maintaining stable weights throughout the lifecycle. Our modeling approach reveals the limitations inherent in bonds and underscores the merits of international stocks for achieving long-term wealth appreciation and real capital preservation. The optimal age-based investment policy delivers substantial welfare gains relative to conventional age-based stock-bond strategies.

Our findings should be of broad interest to regulators defining QDIA-eligible strategies, investment management companies designing retirement funds, and plan sponsors shaping DC plan menus and default investments. These stakeholders may be uneasy with the optimal strategy's lack of perceived safety, as its consistently high equity allocation leads to a wide range of retirement wealth outcomes and exposes investors to the risk of significant intermediate drawdowns. But these features are present in any lifecycle strategy that targets meaningful capital appreciation. In relative terms, the status quo balanced and target-date funds, with their large bond allocations, carry the potential for even larger drawdowns in real terms. These funds also expose investors to greater risk of exhausting their savings. Our results, as a whole, do not suggest that our optimal strategy is safe; they merely suggest that it is safer than the viable alternatives. The optimal strategy's overall risk-return profile thus provides a compelling case for moving beyond the status quo and rethinking traditional approaches to retirement investing.

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Table I: Empirical properties of real returns for bonds and international stocks

The table summarizes the empirical properties of real returns for bonds and international stocks. The underlying data are a monthly panel of real asset class returns for 39 developed countries covering the period from 1890 to 2023. The dataset formation details are provided in Section 3. Panel A reports the annualized mean and standard deviation of real returns for each asset class. Panel B reports variance ratios for each asset class at horizons of one, ten, 20, and 30 years. Panel C reports the correlation of the log return for each asset class with the log return for domestic stocks (based on monthly and 30-year returns) and with log inflation (based on 30-year returns). The statistics reported in Panels B and C are based on a bootstrap simulation. The statistics are estimated from a sample that excludes data for Germany in 1923.

	Asset class	
Measure	Bonds	International stocks
Panel A: Moments of annualized real returns		
Mean (%)	0.95	7.03
Standard deviation (%)	9.51	23.26
Panel B: Variance ratios		
VR(1)	1.00	1.00
VR(10)	2.09	0.88
VR(20)	2.26	0.80
VR(30)	2.30	0.75
Panel C: Log real return correlations		
Correlation with domestic stocks (monthly returns)	0.21	0.33
Correlation with domestic stocks (30-year returns)	0.45	0.34
Correlation with inflation (30-year returns)	−0.78	−0.01

Table II: Sample coverage and summary statistics

For each developed country, the table reports the sample period start date, the sample period end date, and summary statistics (i.e., geometric mean return and standard deviation of return) of monthly real returns for domestic stocks, international stocks, bonds, and bills. The development classification and sample formation criteria are described in the internet appendix.

			Asset class returns							
			Domestic stocks		International stocks		Bonds		Bills	
Country	Sample start	Sample end	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev
Denmark	1890:01	2023:12	0.46	3.58	0.39	3.93	0.20	1.89	0.17	0.73
France	1890:01	2023:12	0.26	5.40	0.45	6.58	−0.08	2.28	−0.16	1.74
Germany	1890:01	2023:12	0.25	8.29	0.56	10.20	−0.14	45.61	0.15	0.86
United Kingdom	1890:01	2023:12	0.41	4.25	0.43	4.09	0.13	1.98	0.06	0.87
United States	1890:01	2023:12	0.52	5.01	0.34	3.85	0.12	1.76	0.06	0.61
Canada	1891:01	2023:12	0.48	4.26	0.44	3.48	0.18	1.66	0.11	0.57
New Zealand	1896:01	2023:12	0.47	3.66	0.44	4.06	0.13	1.84	0.14	0.58
Belgium	1897:01	2023:12	0.21	5.03	0.39	4.57	0.02	1.79	−0.04	1.13
Australia	1901:01	2023:12	0.57	3.96	0.43	3.75	0.14	1.73	0.06	0.53
Sweden	1910:01	2023:12	0.47	4.85	0.45	4.15	0.14	1.86	0.08	0.96
Netherlands	1914:01	2023:12	0.41	5.19	0.41	4.39	0.13	1.73	0.01	0.80
Norway	1914:01	2023:12	0.37	4.62	0.44	4.19	0.13	1.73	0.02	0.85
Switzerland	1914:01	2023:12	0.38	4.30	0.37	4.47	0.14	1.41	0.02	0.61
Austria	1920:01	2023:12	0.22	7.27	0.57	12.79	−0.38	4.51	−0.46	3.68
Czechoslovakia	1922:05	1945:05	−0.14	6.56	0.41	6.24	0.69	3.28	0.34	3.06
Chile period I	1927:01	1970:12	0.13	6.15	0.60	8.55	−0.92	3.38	−0.86	2.34
Japan	1930:01	2023:12	0.31	6.59	0.51	15.92	−0.18	3.40	−0.32	2.60
Italy	1931:01	2023:12	0.19	7.38	0.43	12.98	−0.14	2.56	−0.25	1.68
Portugal	1934:01	2023:12	0.14	7.82	0.44	4.10	0.02	2.81	−0.06	1.34
Ireland	1936:01	2023:12	0.45	4.77	0.46	4.08	0.16	2.40	0.01	0.59
Argentina	1947:02	1966:12	−0.18	8.53	0.64	15.33	−1.66	2.84	−1.56	2.73
Spain	1959:01	2023:12	0.28	5.58	0.39	4.25	0.14	2.22	0.00	0.69
Finland	1969:01	2023:12	0.73	6.24	0.41	4.35	0.24	2.26	0.03	0.47
Greece	1981:02	2023:12	0.48	10.23	0.55	4.77	0.30	5.41	0.14	1.29
Luxembourg	1982:01	2023:12	0.55	5.75	0.57	4.51	0.28	1.88	0.10	0.57
Singapore	1998:07	2023:12	0.44	5.74	0.29	4.09	0.14	2.04	−0.04	0.48

(Continued on next page)

Table II (Continued)

			Asset class returns							
			Domestic stocks		International stocks		Bonds		Bills	
Country	Sample start	Sample end	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev
Hungary	1999:02	2023:12	0.35	6.52	0.26	4.21	0.17	3.49	0.09	0.53
Poland	1999:06	2023:12	0.26	6.20	0.21	3.78	0.22	2.92	0.12	0.53
Slovakia	2000:01	2023:12	0.17	5.10	−0.02	4.26	0.24	2.96	−0.12	0.62
Czech Republic	2000:05	2023:12	0.77	6.92	0.01	4.23	0.08	2.45	−0.10	0.63
South Korea	2000:11	2023:12	0.63	6.18	0.34	3.93	0.25	2.02	0.06	0.36
Mexico	2001:08	2023:12	0.62	4.94	0.44	3.72	0.29	2.67	0.14	0.40
Iceland	2002:01	2023:12	0.02	7.27	0.33	4.80	0.16	3.16	0.14	0.54
Chile period II	2010:01	2023:12	−0.01	4.89	0.70	3.99	−0.05	1.93	−0.02	0.43
Israel	2010:01	2023:12	0.01	5.07	0.68	3.63	0.28	2.14	0.09	0.87
Slovenia	2010:01	2023:12	0.41	4.75	0.72	3.79	0.09	2.69	−0.10	0.80
Türkiye	2010:02	2023:12	0.36	7.42	1.11	6.49	−0.79	6.92	−0.47	1.77
Latvia	2016:01	2023:12	0.47	4.42	0.40	4.09	−0.51	2.23	−0.36	0.82
Lithuania	2018:01	2023:12	0.02	3.85	0.26	4.57	−0.82	2.76	−0.46	0.80
Colombia	2020:01	2023:12	−0.79	9.02	0.46	4.75	−0.44	4.27	−0.08	0.51

Table III: Economic value of alternative investment plans

The table reports the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal asset allocation policy conditional on household age (Panel A) and four benchmark investment strategies (Panel B). The weights for the TDF strategy and the optimal age-based strategy change over the lifecycle following the glide paths shown in Figures 1 and 3, respectively. The final column of the table reports equivalent savings rates to quantify relative economic value in pairwise comparisons of the optimal age-based strategy with each alternative strategy. Each comparison is based on expected household utility over retirement consumption and bequest across 1,000,000 bootstrap simulations. The household's savings rate for the base strategy (i.e., the optimal age-based asset allocation strategy) in the pre-retirement period is 10%. The equivalent savings rate is the household's savings rate for the alternative strategy that equates the expected utility from retirement consumption and bequest for the base and alternative strategies.

	Asset class weights				
Strategy	Domestic stocks	International stocks	Bonds	Bills	Equivalent savings rate
Panel A: Optimal age-based strategy					
Optimal Age-Based	[24%, 41%]	[47%, 76%]	[0%, 0%]	[0%, 27%]	—
Panel B: Alternative strategies					
Bills	0%	0%	0%	100%	56.71%
Domestic Stocks	100%	0%	0%	0%	16.42%
Balanced	60%	0%	40%	0%	19.44%
TDF	[10%, 54%]	[7%, 36%]	[10%, 73%]	[0%, 10%]	16.27%

Table IV: Retirement saving outcomes

The table summarizes the distributions of real wealth at retirement (Panel A), the real income replacement rate (Panel B), and real wealth at death (Panel C) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. For each asset allocation strategy, the table reports the mean, standard deviation, and distribution percentiles of each measure of investment performance. Real wealth at retirement and real wealth at death are reported in millions of 2022 USD.

	Moments		Percentiles								
Strategy	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Panel A: Wealth at retirement ($MM)											
Bills	0.27	0.21	0.00	0.03	0.06	0.13	0.23	0.36	0.53	0.65	0.98
Domestic Stocks	1.02	1.75	0.00	0.07	0.14	0.30	0.61	1.18	2.14	3.13	6.79
Balanced	0.71	1.94	0.00	0.06	0.12	0.26	0.48	0.85	1.40	1.91	3.73
TDF	0.77	1.30	0.00	0.09	0.16	0.31	0.55	0.93	1.49	2.00	3.78
Optimal Age-Based	1.06	1.40	0.00	0.13	0.22	0.41	0.74	1.27	2.11	2.91	5.90
Panel B: Income replacement rate											
Bills	0.70	1.27	0.00	0.43	0.49	0.57	0.66	0.78	0.93	1.06	1.73
Domestic Stocks	1.18	1.63	0.00	0.50	0.59	0.73	0.95	1.31	1.90	2.52	4.82
Balanced	0.99	1.71	0.00	0.50	0.57	0.70	0.86	1.10	1.45	1.79	3.07
TDF	1.03	1.55	0.00	0.52	0.60	0.73	0.90	1.15	1.51	1.86	3.16
Optimal Age-Based	1.24	1.55	0.00	0.59	0.68	0.83	1.05	1.39	1.92	2.44	4.54
Panel C: Wealth at death ($MM)											
Bills	0.08	0.14	0.00	0.00	0.00	0.00	0.02	0.11	0.23	0.34	0.64
Domestic Stocks	2.61	14.71	0.00	0.00	0.00	0.10	0.59	1.93	5.29	9.76	31.65
Balanced	1.10	8.35	0.00	0.00	0.00	0.07	0.35	0.98	2.29	3.83	10.65
TDF	0.72	5.21	0.00	0.00	0.00	0.04	0.27	0.73	1.57	2.47	6.00
Optimal Age-Based	2.66	9.96	0.00	0.00	0.07	0.35	0.96	2.38	5.43	9.17	27.20

Table V: Portfolio drawdowns

The table summarizes the distributions of the maximum portfolio drawdown during the pre-retirement period (Panel A) and the maximum portfolio drawdown during the post-retirement period (Panel B) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. For each asset allocation strategy and drawdown period, the table reports the mean, standard deviation, and distribution percentiles of the maximum portfolio drawdown.

	Moments		Percentiles								
Strategy	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Panel A: Working-period drawdown											
Bills	0.42	0.26	0.04	0.09	0.13	0.22	0.37	0.57	0.84	0.96	1.00
Domestic Stocks	0.67	0.16	0.32	0.41	0.46	0.55	0.66	0.78	0.91	0.96	0.99
Balanced	0.54	0.19	0.23	0.29	0.32	0.41	0.50	0.66	0.85	0.92	0.99
TDF	0.52	0.16	0.22	0.28	0.32	0.41	0.51	0.61	0.74	0.81	0.94
Optimal Age-Based	0.55	0.13	0.24	0.31	0.38	0.48	0.56	0.61	0.69	0.77	0.91
Panel B: Retirement-period drawdown											
Bills	0.46	0.30	0.00	0.03	0.09	0.22	0.42	0.69	0.94	0.98	1.00
Domestic Stocks	0.62	0.21	0.00	0.27	0.35	0.49	0.63	0.77	0.91	0.96	0.99
Balanced	0.50	0.23	0.00	0.18	0.24	0.34	0.47	0.66	0.85	0.92	0.98
TDF	0.40	0.23	0.00	0.11	0.15	0.23	0.35	0.54	0.76	0.89	0.99
Optimal Age-Based	0.47	0.17	0.00	0.18	0.24	0.36	0.51	0.58	0.65	0.73	0.88

Table VI: Optimal asset allocation policy conditional on the aggregate price-dividend ratio

The table reports the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal fixed-weight asset allocation policy (Panel A) and the optimal asset allocation policy conditional on the aggregate price-dividend ratio (Panel B).

		Optimal asset class weights			
Market state	Aggregate P_t/D_t	Domestic stocks	International stocks	Bonds	Bills
Panel A: Optimal fixed-weight strategy					
All	$[0, \infty)$	33%	67%	0%	0%
Panel B: Optimal strategy conditional on aggregate price-dividend ratio					
Low P_t/D_t	$[0, 18.76]$	65%	35%	0%	0%
2	$(18.76, 23.47]$	28%	72%	0%	0%
3	$(23.47, 29.94]$	30%	70%	0%	0%
4	$(29.94, 43.67]$	31%	69%	0%	0%
High P_t/D_t	$(43.67, \infty)$	16%	75%	9%	0%

Table VII: Optimal fixed-weight asset allocation policies under alternative design parameters
Panel A reports the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal fixed-weight asset allocation policy. The subsequent panels report the optimal fixed-weight asset allocation policies under alternative design assumptions, as described in the table. The last column shows the amount of borrowing as a percentage of wealth for the specifications in which leverage is allowed.

	Optimal asset class weights				
Description	Domestic stocks	International stocks	Bonds	Bills	Borrowing (% of wealth)
Panel A: Optimal fixed-weight strategy					
Optimal fixed-weight strategy	33%	67%	0%	0%	N/A
Panel B: Alternative samples					
Post-World War II sample	31%	69%	0%	0%	N/A
Countries with large populations	27%	73%	0%	0%	N/A
Countries with large market cap-to-GDP ratios	27%	73%	0%	0%	N/A
Ex-United States sample	45%	55%	0%	0%	N/A
Panel C: Average block length					
Average block length of 60 months	33%	67%	0%	0%	N/A
Average block length of 240 months	33%	67%	0%	0%	N/A
Panel D: Risk aversion					
Risk aversion parameter $\gamma = 0.5$	32%	68%	0%	0%	N/A
Risk aversion parameter $\gamma = 1.0$	35%	65%	0%	0%	N/A
Risk aversion parameter $\gamma = 2.0$	35%	65%	0%	0%	N/A
Risk aversion parameter $\gamma = 5.0$	33%	67%	0%	0%	N/A
Risk aversion parameter $\gamma = 7.5$	33%	67%	0%	0%	N/A
Risk aversion parameter $\gamma = 10.0$	33%	67%	0%	0%	N/A
Panel E: Bequest					
No bequest motive ($\theta = 0$)	33%	67%	0%	0%	N/A
Utility only from bequest ($\theta = \infty$)	34%	66%	0%	0%	N/A
Panel F: Household utility specification					
Consumption scaled by household size	33%	67%	0%	0%	N/A
Subjective discount factor $\delta = 0.98^{1/12}$	33%	67%	0%	0%	N/A
Panel G: Withdrawal strategy					
3% rule ($r_w = 3\%$)	34%	66%	0%	0%	N/A
5% rule ($r_w = 5\%$)	34%	66%	0%	0%	N/A
4% of current account value	34%	66%	0%	0%	N/A
Panel H: Retirement age					
Retirement at age 62	33%	67%	0%	0%	N/A
Retirement at age 67	33%	67%	0%	0%	N/A
Retirement at age 70	33%	67%	0%	0%	N/A

(Continued on next page)

Table VII (Continued)

Description	Optimal asset class weights: Domestic stocks	International stocks	Bonds	Bills	Borrowing (% of wealth)
Panel I: Contribution rules					
Contribution rate $r_c = 5\%$	33%	67%	0%	0%	N/A
Contribution rate $r_c = 15\%$	33%	67%	0%	0%	N/A
Lower income limit $Y_{min} = \$0$	33%	67%	0%	0%	N/A
Lower income limit $Y_{min} = \$45,000$	33%	67%	0%	0%	N/A
Panel J: Household type					
Single female	33%	67%	0%	0%	N/A
Single male	33%	67%	0%	0%	N/A
Both female	33%	67%	0%	0%	N/A
Both male	33%	67%	0%	0%	N/A
Panel K: Investor type					
Low initial income and low human capital	33%	67%	0%	0%	N/A
Low initial income and high human capital	33%	67%	0%	0%	N/A
High initial income and low human capital	33%	67%	0%	0%	N/A
High initial income and high human capital	32%	68%	0%	0%	N/A
Panel L: Correlation between persistent earnings shocks and domestic stock returns					
Income-domestic stock correlation of 0.1	30%	70%	0%	0%	N/A
Income-domestic stock correlation of 0.2	27%	73%	0%	0%	N/A
Income-domestic stock correlation of 0.3	24%	76%	0%	0%	N/A
Income-domestic stock correlation of 0.4	21%	79%	0%	0%	N/A
Income-domestic stock correlation of 0.5	18%	82%	0%	0%	N/A
Panel M: Leverage					
Borrowing spread of 6.50% (high)	33%	67%	0%	0%	0%
Borrowing spread of 1.40% (medium)	34%	66%	0%	0%	55%
Borrowing spread of 0.37% (low)	28%	57%	15%	0%	100%

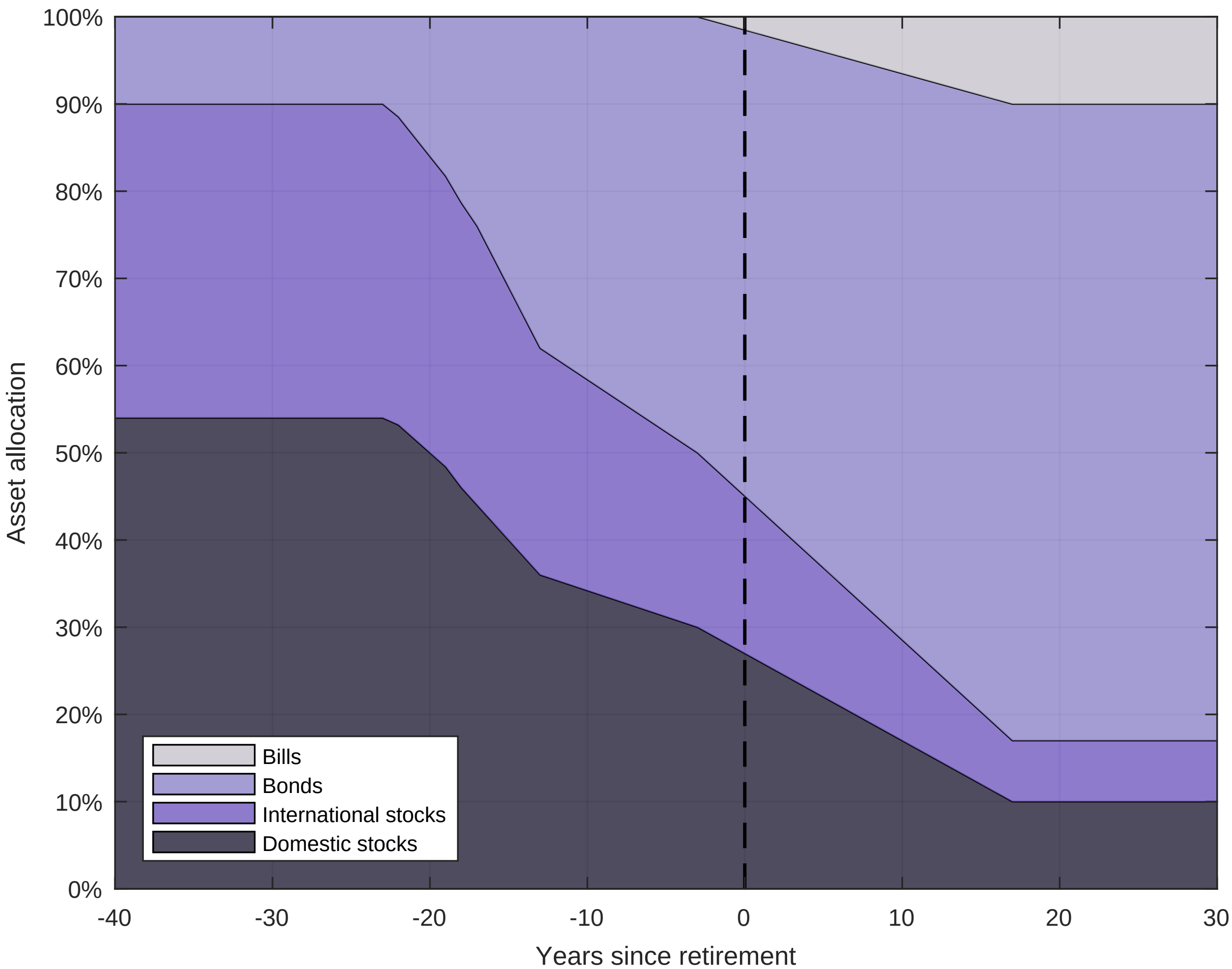

Figure 1. Glidepath weights for the target-date fund. The figure shows the asset allocation of the target-date strategy as a function of time since retirement. The strategy invests in domestic stocks, international stocks, bonds, and bills.

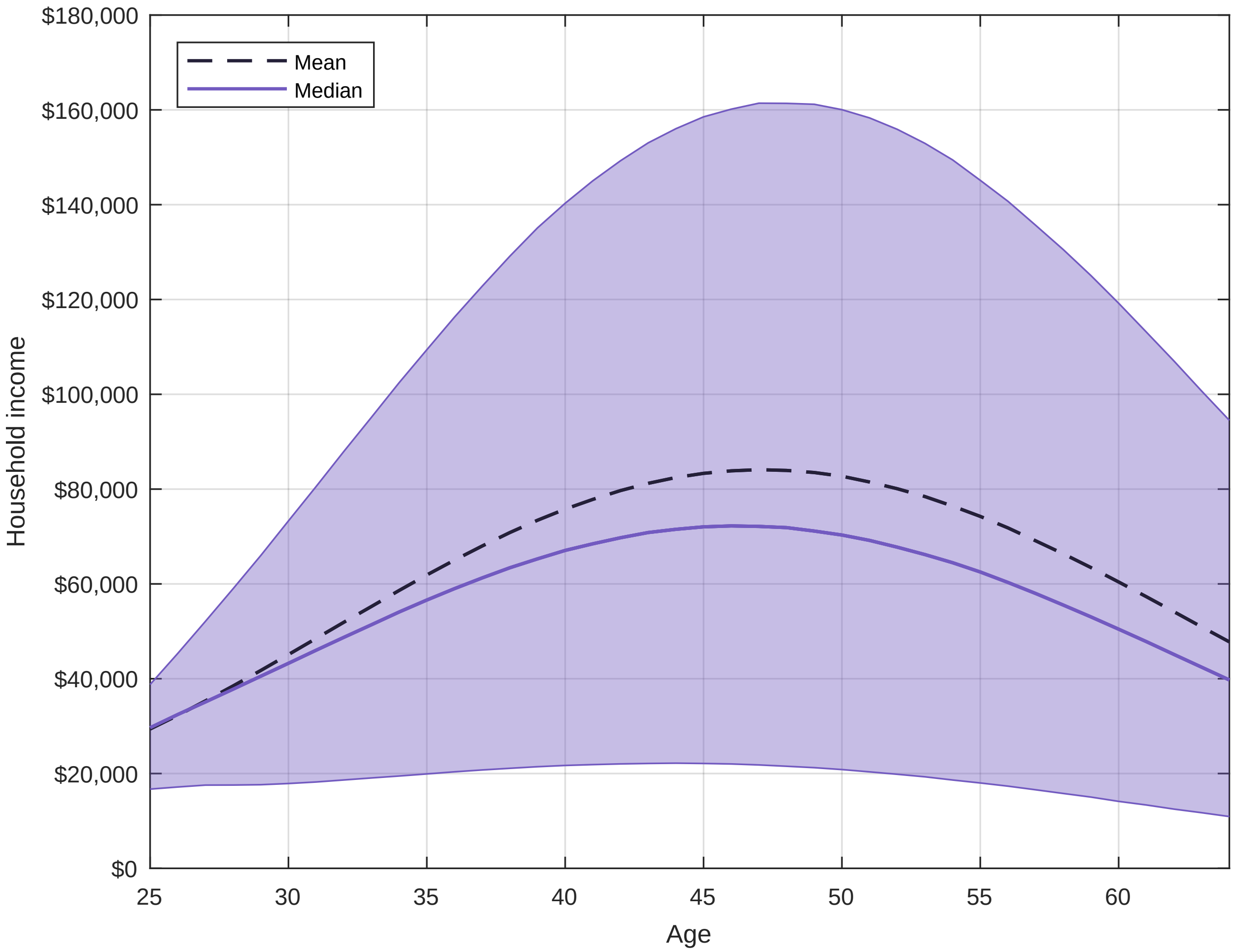

Figure 2. Distribution of household income. The figure shows the distribution of real household income across 1,000,000 bootstrap simulations in 2022 USD as a function of age. Household income is stochastic and follows the process estimated by Guvenen, Karahan, Ozkan, and Song (2021) with an initial income parameter of $z_0^i = 0$ and human capital parameters of $(\alpha^i, \beta^i) = (0, 0)$. The solid (dashed) line corresponds to the median (mean) household income as a function of age. The shaded region covers the 10th through 90th percentiles of the distribution.

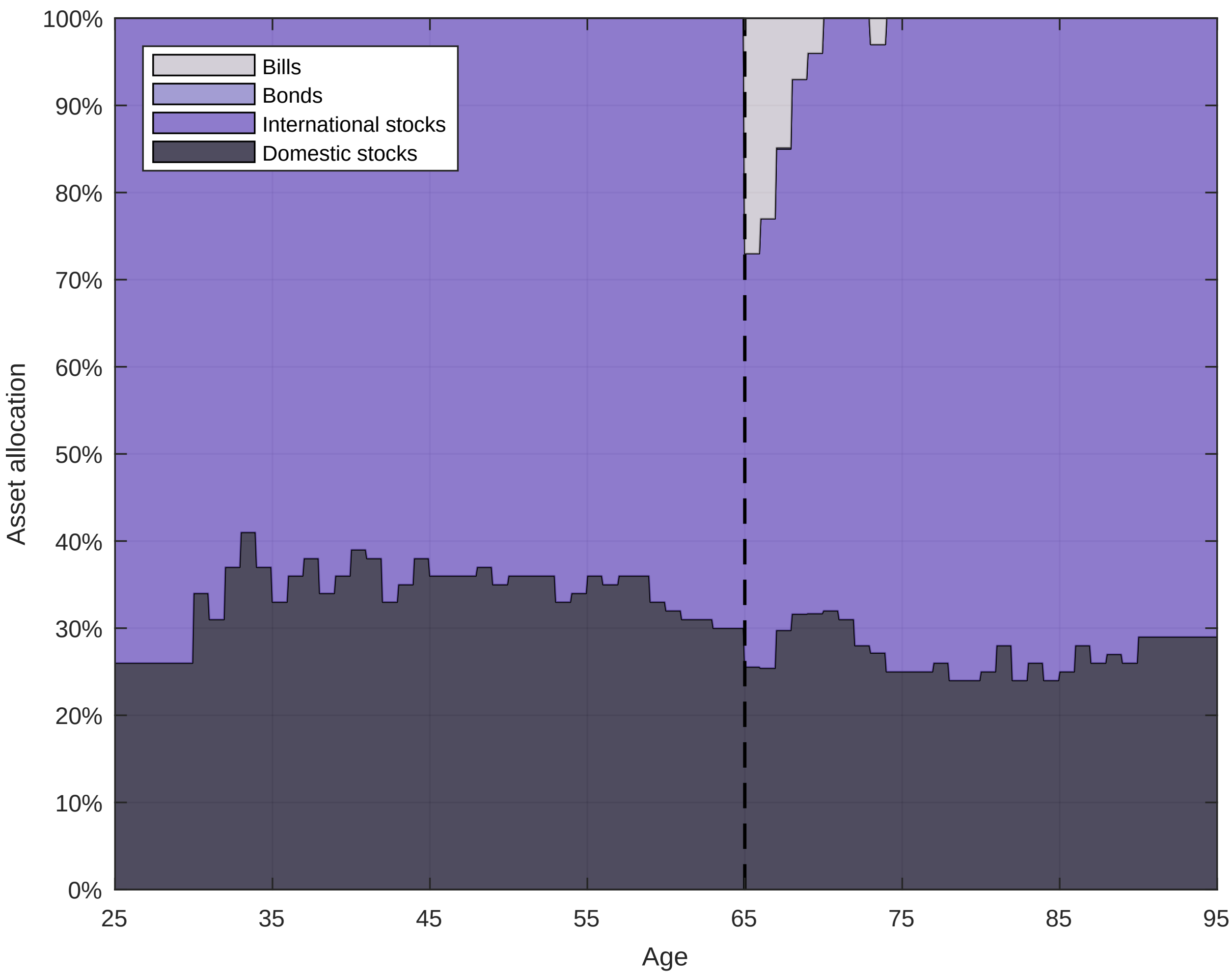

Figure 3. Optimal age-based asset allocation policy. The figure shows the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal asset allocation policy conditional on household age. The vertical dashed line indicates the household's exogenous retirement age.

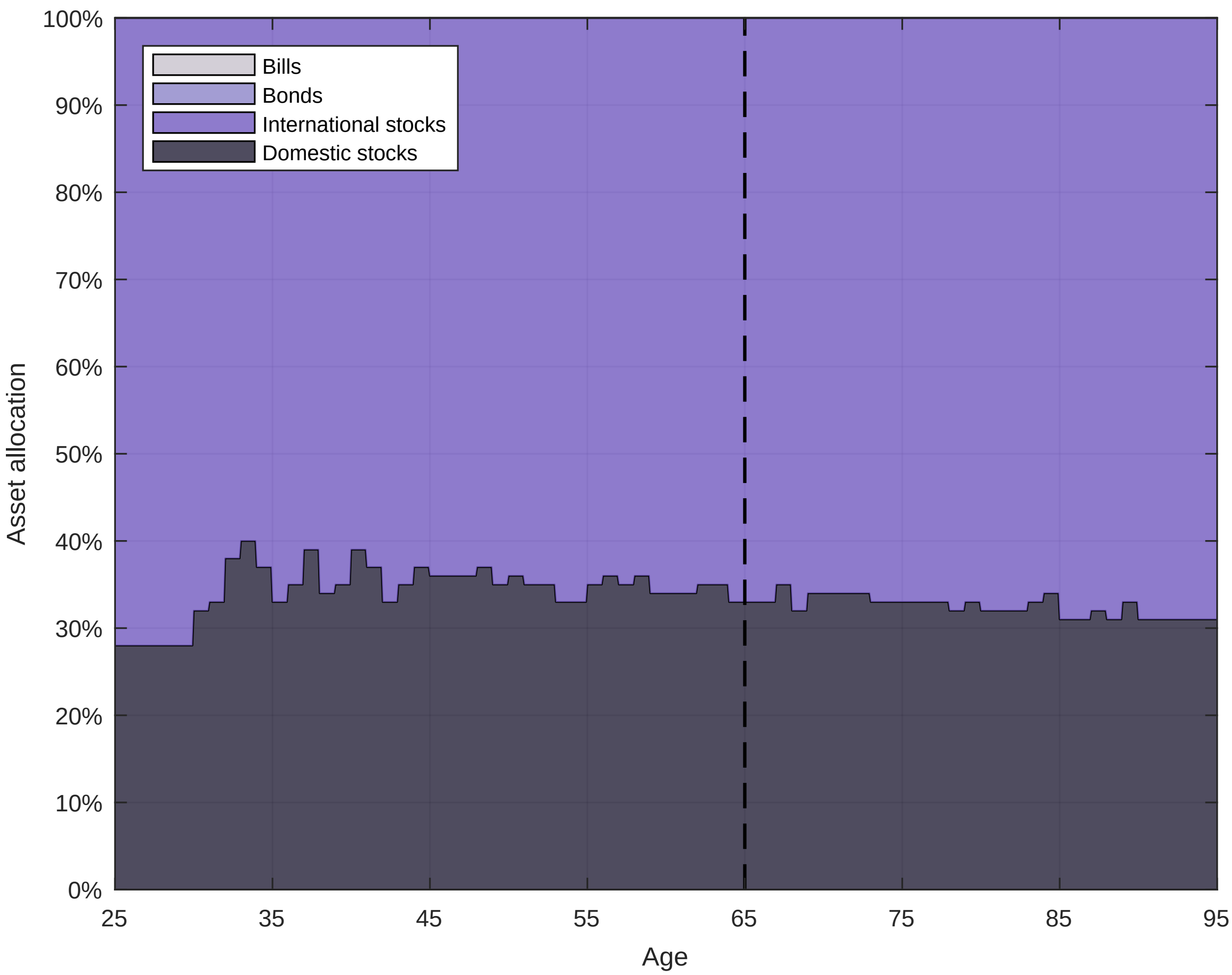

Figure 4. Optimal age-based asset allocation policy with a 4% of current wealth withdrawal rule. The figure shows the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal asset allocation policy conditional on household age. The household's monthly withdrawal strategy in retirement is equal to 4% (annualized) of current wealth. The vertical dashed line indicates the household's exogenous retirement age.

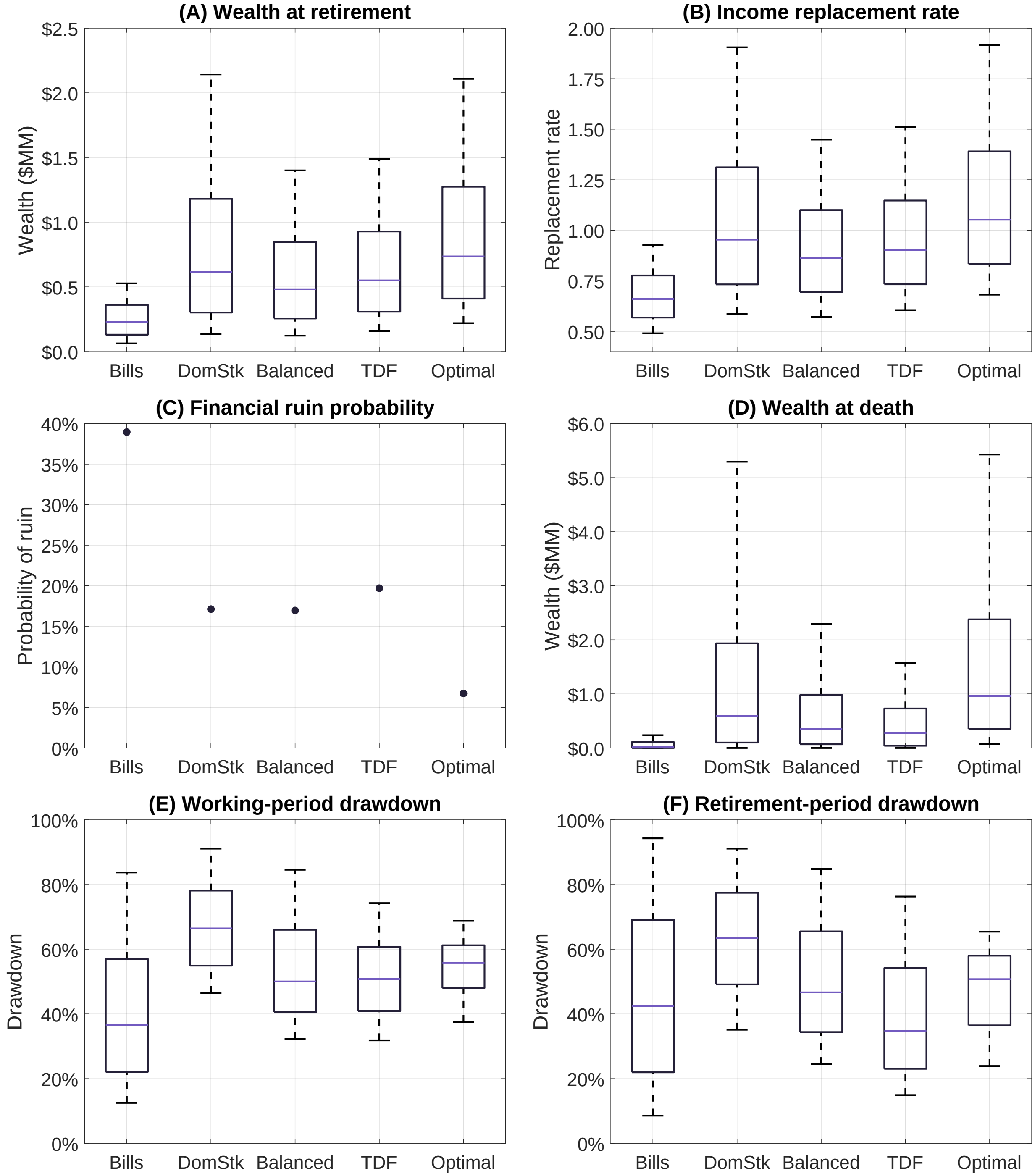

Figure 5. Measures of investment performance. The figure summarizes the distribution of real wealth at retirement (Panel A), the distribution of the real income replacement rate (Panel B), the probability of financial ruin (Panel C), the distribution of real wealth at death (Panel D), the distribution of the working-period drawdown (Panel E), and the distribution of the retirement-period drawdown (Panel F) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. In each box-and-whiskers plot, the middle line corresponds to the median, the box covers the interquartile range, and the whiskers cover the 10th through 90th percentiles.

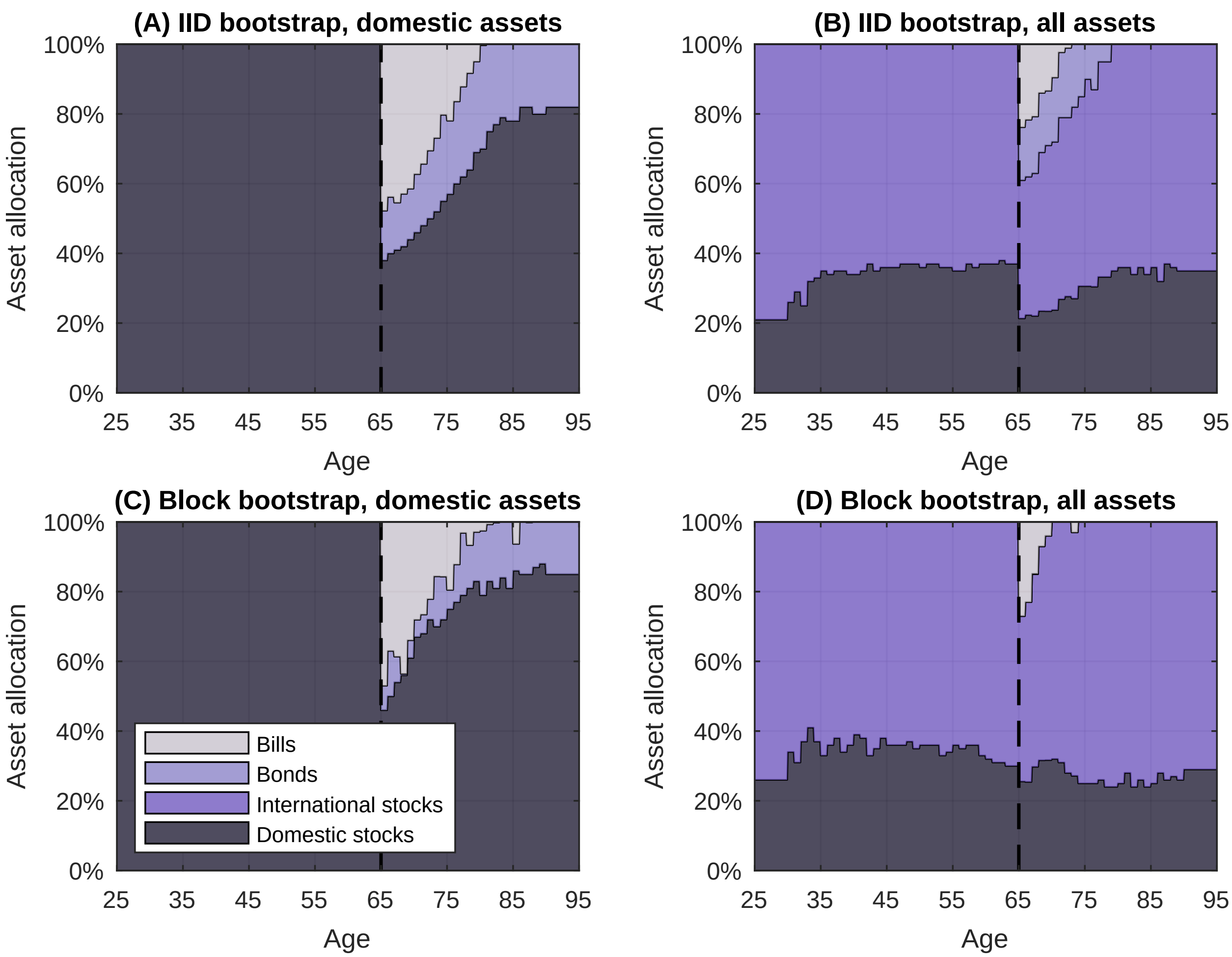

Figure 6. Optimal age-based asset allocation policy: Alternative bootstrap designs and investment opportunity sets. The figure shows the optimal asset allocation policy conditional on household age for various underlying bootstrap specifications and investment opportunity sets. The simulations used to estimate the optimal strategies in Panels A and B are based on an IID bootstrap, and those used to estimate the optimal strategies in Panels C and D are based on a block bootstrap with an average block length of 120 months. The strategies in Panels A and C invest in domestic stocks, bonds, and bills. The strategies in Panels B and D add international stocks. In each panel, the vertical dashed line indicates the household's exogenous retirement age.

Internet Appendix

"Beyond the Status Quo: A Critical Assessment of Lifecycle Investment Advice"

A Data appendix

This appendix describes our development classification approach, data sources, calculations of asset class returns, special data issues, and dataset validation. Section A.1 outlines our development classification and data sources used to compute asset class returns. Sections A.2 to A.5 provide details on the calculations of returns for domestic stocks, international stocks, government bonds, and government bills, respectively, and special data issues related to each asset class. Section A.6 presents data construction details for other variables. Section A.7 compares our data on stock and bond returns with data from alternative sources.

A.1 Development classification and data sources

We follow Anarkulova, Cederburg, and O'Doherty (2022) to classify countries as developed. We classify a given country as developed early in the sample period if its agricultural labor share is less than 50% based on evidence about labor patterns from the economics literature [e.g., Kuznets (1973)]. Beginning with the formation of the Organisation for European Economic Co-operation (OEEC) in 1948, we use membership in the OEEC and the Organisation for Economic Co-operation and Development (OECD) to identify development dates.

To form a balanced panel, a given developed country is not included in our sample until its government issues ten-year bonds. Sample eligibility postdates development for several countries on this basis. The sample eligibility date is the latest of 1890 (i.e., the sample period start date for our study), the country development year, and the year in which the country first issued long-term bonds.

Table A.I displays the development date, reason for classification, sample eligibility date, and data coverage for each country. In three instances, a previously developed country is reclassified as developing. These instances occur in Argentina, Chile, and Czechoslovakia, and each reclassification results from substantial changes in governments and markets in these countries. Chile, the Czech Republic, and Slovakia reenter the sample with membership in the OECD. We include the early periods in these countries to avoid survivor bias.

For some countries, we have missing data at the beginning of the eligible period. Returns on a diversified domestic stock index are the binding data constraint in each of these instances. No country has data gaps in the middle or at the end of its series.

The primary source of data for our study is the GFDatabase from Global Financial Data (GFD). The database contains long time series of total return indexes, price indexes, dividend-price ratios, and total market capitalization for stocks; yields for ten-year government bonds and short-term bills; consumer price indexes; and exchange rates for a broad set of countries. Table A.II reports the data series that we use to compute monthly stock, bond, and bill returns for each country. As described in the footnotes to Table A.II, we supplement the data from GFD with data from other sources.

A.2 Domestic stocks

The GFDatabase contains data for total return indexes, price indexes, and dividend-price ratios. It includes stock market indexes that are created and calculated by stock exchanges (e.g., the Tokyo Stock

Price Index from the Tokyo Stock Exchange), by well-known index providers (e.g., the S&P 500 Index), or by GFD directly from original source documents. Multiple stock indexes are available in the database for some countries and periods. We select a single index in these cases by considering the breadth of market coverage and the length of historical coverage. We use a total return index whenever one is available, and we otherwise use a price index and a dividend-price ratio to calculate returns.

For sample months in which a total return index is available, we calculate the monthly nominal return,

$$R_{i,t}^{Nominal\ stocks} = \frac{I_{i,t}^{Total}}{I_{i,t-1}^{Total}}, \tag{A1}$$

where $I_{i,t}^{Total}$ is the total return index for country i at the end of month t and $R_{i,t}^{Nominal\ stocks}$ is the gross nominal return for country i in month t. If no total return index is available, we use price index and dividend-price ratio data to calculate returns. We assume that the annual dividend reflected by the reported dividend-price ratio is paid equally across months in the year. If $I_{i,t}^{Price}$ is the price index and $\hat{D}_{i,t}$ is the estimated dividend (appropriately scaled to the level of the price index) for country i in month t, then we calculate the monthly nominal return,

$$R_{i,t}^{Nominal\ stocks} = \frac{I_{i,t}^{Price} + \hat{D}_{i,t}}{I_{i,t-1}^{Price}}. \tag{A2}$$

Nominal returns reflect diversified investments in a broad country-level index. To calculate real returns, we first calculate gross inflation,

$$\Pi_{i,t} = \frac{I_{i,t}^{CPI}}{I_{i,t-1}^{CPI}}, \tag{A3}$$

where $I_{i,t}^{CPI}$ is the consumer price index (CPI) for country i at the end of month t. We then calculate the gross real return on domestic stocks,

$$R_{i,t}^{Domestic\ stocks} = \frac{R_{i,t}^{Nominal\ stocks}}{\Pi_{i,t}}. \tag{A4}$$

This return calculation produces real returns that are denominated in the local currency of country i.

A.2.1 Data issues related to domestic stocks

Our treatments of special data issues mirror those in Anarkulova, Cederburg, and O'Doherty (2022) with minor exceptions. Details on the data adjustments required to compute nominal and real stock returns for our developed country sample are available in Anarkulova, Cederburg, and O'Doherty (2022) and the corresponding internet appendix. Additional adjustments are described below.

We measure returns that are denominated in the primary home currency with one exception. Our real returns for Germany are denominated in gold marks (rather than paper marks) for the 1917 to 1923 period. Extraordinary hyperinflation during this period complicates the calculation of real returns based on nominal returns in paper marks, and the GFDatabase reports a series of stock market returns denominated in gold marks.

The internet appendix for Anarkulova, Cederburg, and O'Doherty (2022) outlines the smoothing procedures used to fill gaps in return series for short periods in a few sample countries. In addition to those cases, we apply a smoothing procedure to convert five- and seven-month return data for Austria over the period from January 1920 to February 1922 and quarterly return data for Belgium over the period from May 1919 to January 1926 into time series of monthly returns. In particular, we make the assumption of constant monthly returns within each period. We apply a similar procedure to quarterly return data for Switzerland from February 1914 to July 1914 and from August 1916 to January 1921. Czechoslovakia is missing return data for July 1921. We estimate returns for July 1921 and August 1921 using price index data for June 1921 and August 1921 under the assumption of a constant return for July and August of 1921.

One difference between our sample construction approach and the one in Anarkulova, Cederburg, and O'Doherty (2022) relates to the handling of multi-month return observations associated with stock market disruptions and closures. There are 35 instances in which stock exchanges closed for extended periods, typically as the result of a major war, political revolution, or banking crisis. Investors tend to earn negative real returns in these periods, such that omitting countries or periods because of these stock return data gaps induces an easy data bias. Table A.III reports cases of exchange closures or heavily restricted trading during our sample period along with the corresponding nominal and real returns. The bootstrap procedure in Anarkulova, Cederburg, and O'Doherty (2022) treats each of these events as a single return observation covering a multi-month period. This treatment reflects that most investors would have been unable to trade during these periods, such that they could only wait for the eventual realizations of the longer-period returns. Such treatment is not ideal for our multi-asset analysis, which requires a balanced panel of monthly asset returns for each country. At the same time, we need the data to reflect the economic outcomes of stock market investors.

In our current approach to handling multi-month returns, we take the perspective of an investor in a hypothetical fund attempting to track the market index for a given country. Although this investor could not directly liquidate her stock holdings via exchange trades during times of market closure, she could sell her shares in the hypothetical fund. The fund's managers, in turn, could either rely on black market data for valuation purposes or produce an estimate of the historical event's impact on asset prices at the beginning of the closure period. Based on this perspective, we apply one of two approaches to handling multi-month returns:

1. For events during which GFD provides black market prices, we use these values to estimate stock market index returns.

2. For events without corresponding data in GFD, we assign the total multi-month real return to the first monthly observation and zero real return to the remaining monthly observations.

The three exceptions to this general approach correspond to Austria's 113-month return from July 1939 to November 1948, Switzerland's 24-month return from August 1914 to July 1916, and Czechoslovakia's 26-month return from April 1943 to May 1945. For Austria, GFD reports limited black market data in January 1943, April 1946, and from November 1946 to November 1948. We use these intermittent values and assign the remaining part of the total real return to July 1939. Similarly for Switzerland, we use GFD's black market data in January 1916 and July 1916 and assign the remaining part of the total real return to August 1914. For Czechoslovakia, the April 1943 to May 1945 period corresponds to an episode that starts with severe trading restrictions and price controls and ends with the permanent stock exchange closure in Prague on May 5, 1945. For this period, we assign a terminal nominal return of −90.00% to May 1945 and zero nominal returns to the other months. This treatment of Czechoslovakia's multi-month return is consistent with the economic experience of investors over this period, as detailed in Anarkulova, Cederburg, and O'Doherty (2022).

A.3 International stocks

We calculate real returns on a portfolio of international stocks from the perspective of an investor in a developed country. For each country, the international stock portfolio is a weighted investment across all developed stock markets excluding the local stock market. The international stock portfolio is value weighted by total market capitalization, and the returns are expressed in the domestic currency such that they reflect the exchange rate risk incurred by investing in assets denominated in foreign currencies.

The return calculation for international stocks uses the gross nominal stock market returns described in Section A.2. We convert the nominal return for each country $j \neq i$ into a real return denominated in the domestic currency of country i and calculate the weighted average across countries $j \neq i$,

$$R_{i,t}^{International\ stocks} = \sum_{j \neq i} w_{j,t-1} \frac{R_{j,t}^{Nominal\ stocks}}{\Pi_{i,t}} \left(\frac{E_t^{i,j}}{E_{t-1}^{i,j}} \right), \tag{A5}$$

where $E_t^{i,j}$ is the exchange rate at the end of month t expressed in country i's currency per country j's currency, $w_{j,t-1}$ is country j's weight in the international stock portfolio in month t,

$$w_{j,t-1} = \frac{M_{j,t-1}}{\sum_{j \neq i} M_{j,t-1}}, \tag{A6}$$

and $M_{j,t-1}$ is the total market capitalization for the stock market in country j at the end of month $t-1$ expressed in USD.

A.4 Bonds

We calculate bond returns using monthly data on bond yields. For comparability across countries and periods, we focus on ten-year government bonds. The GFDatabase has variables for ten-year bond yields for most countries and periods in our sample, and we supplement these data to achieve full data coverage.

We first estimate ten-year bond prices given bond yields. We assume that each bond has exactly ten years to maturity, semiannual coupons, and a coupon rate equal to the greater of the bond yield and zero at the end of month $t-1$. We then reprice the bond at the end of month t given the month-t yield and the one month shorter maturity. We calculate the gross nominal return,

$$R_{i,t}^{Nominal\ bonds} = \frac{P_{i,t}}{P_{i,t-1}}, \tag{A7}$$

where $P_{i,t}$ is the calculated dirty bond price (i.e., inclusive of accrued interest) for country i at the end of month t. Finally, we calculate the gross real bond return,

$$R_{i,t}^{Bonds} = \frac{R_{i,t}^{Nominal\ bonds}}{\Pi_{i,t}}. \tag{A8}$$

The return calculation requires assumptions about the maturity and the coupon rate of the underlying bond. We validate this calculation in Section A.7 by comparing our calculated returns with returns from Datastream over the period of overlap between the two data samples. Our return calculations are highly correlated with and have similar moments to those from Datastream.

Sections A.4.1 to A.4.7 describe several issues related to the underlying bond yield data.

A.4.1 Bond data availability

For several countries in our sample, there are no ten-year government bonds in circulation at the time the country is initially classified as developed. For example, ten-year government bonds are first issued in Iceland in 1992, Singapore in 1998, Hungary in 1999, Poland in 1999, the Czech Republic in 2000, South Korea in 2000 [Kang, Kim, and Rhee (2005)], Mexico in 2001 [Jeanneau and Verdia (2005)], and Türkiye in 2010.[34] These circumstances create gaps between the development dates and the sample eligibility dates for these countries.

Estonia issued its only domestic bond in 1993, and all tranches were redeemed by 2004.[35] As a result, Estonia is excluded from our sample because the country has no domestic bond data for the developed period.

A.4.2 Data gaps and errors

Table A.IV shows periods over which we are missing monthly bond yields. In these cases, we use a smoothing procedure to fill gaps in the monthly bond return series. The procedure uses the country-level yield data from before and after the missing observations to produce a series of constant monthly returns across a given period.

Because there are no data from GFD or alternative sources, we use the last non-missing yield of 4.33% in June 1944 to fill the data gap in bond yield data for Czechoslovakia from July 1944 to May 1945.

We adjust an apparent error in the GFD bond yield data for Switzerland. The source for the GFD data is the Swiss National Bank. In comparing the GFD data to the Swiss National Bank data, however, the yields match only through December 1941. The Swiss National Bank reports yields of 3.11% in January 1942, 3.14% in February 1942, 3.12% in March 1942, and 3.08% in April 1942. GFD reports yields of 3.14%, 3.12%, and 3.07% for January through March 1942. From April 1942 to December 1990, the GFD data lead the Swiss National Bank data by one month. We adjust the GFD data by entering a 3.11% yield for January 1942 and shifting the original GFD data from January 1942 to November 1990 so that it covers February 1942 to December 1990.

A.4.3 Merging multiple sources

As shown in Table A.II, constructing a series of bond returns for a given country often requires us to combine yield data from multiple sources. We make additional adjustments in linking the data series for several sample countries. The GFD data for Chile end in March 2015, and we use data from Federal Reserve Economic Data (FRED) from April 2015 to December 2023. GFD reports a yield of 2.23% for March 2015, whereas the yields from FRED are 4.34% for March 2015 and 4.49% for April 2015. Merging these data series without adjustment would result in a return calculation of −17.76% for April 2015. This return likely provides a poor characterization of investment outcomes, given the relative stability in yields in the FRED data. To address this issue, we use March 2015 and April 2015 yields from FRED to compute the April 2015 bond return. We make an analogous adjustment for Iceland

[34] See http://www.lanamal.is/asset/12732/special-report-markadsvidskipti_agust-2019.pdf for Iceland, https://eservices.mas.gov.sg/statistics/fdanet/BenchmarkPricesAndYields.aspx for Singapore, https://stats.oecd.org/OECDStat_Metadata/ShowMetadata.ashx?Dataset=GOV_DEBT&Coords=%5BCOU%5D.%5BHUN%5D&ShowOnWeb=true&Lang=en for Hungary, https://www.gov.pl/web/finance/transaction-database for Poland, https://www.cnb.cz/en/financial-markets/treasury-securities-market/government-bonds/ for the Czech Republic, and https://www.tcmb.gov.tr/wps/wcm/connect/EN/TCMB+EN/Main+Menu/Statistics/Markets+Data/Treasury+Auction/ for Türkiye.

[35] See https://www.rahandusministeerium.ee/en/objectivesactivities/state-treasury/financial-reserves-and-liabilities/debt-management.

in March 2004, as well as for Lithuania, Luxembourg, Poland, Portugal, Slovakia, and Türkiye in the corresponding months of 2023 when we merge data from GFD and FRED.

A.4.4 Alternative bond return calculations

Our primary bond return calculations use yield data with an assumption that the coupon rate is equal to the bond yield for a hypothetical new ten-year bond. In the cases described below, we use an alternative approach of separately measuring the capital gain and the coupon income due to data availability. We use data on current yields and coupon rates from the Central Bank of Argentina to infer bond prices for each month end from January 1947 to December 1966. We compute the capital gain based on the change in bond price and add one month of coupon income based on the 3% coupon rate from February 1947 to July 1960 and the 8% coupon rate from August 1960 to December 1966. We use London quotes from the International Center for Finance at Yale for Chile (December 1926 to September 1929) and Czechoslovakia (April 1922 to January 1927). We compute monthly bond returns based on price changes and monthly coupon income at the coupon rate of 4.5% for Chile and 8.0% for Czechoslovakia. Similarly, we compute monthly bond returns based on price changes and monthly coupon income at the coupon rate of 4% for Chile from April 1965 to December 1970. We use bond price data from the Central Bank of Chile over this period.

A.4.5 Bond conversion in Argentina

Argentina issued a 3% bond in 1955. In August 1960, the government allowed for a voluntary conversion of these old bonds to new 8% bonds. The conversion was favorable for bondholders, as they could receive bonds with higher interest payments. According to Duggan (1963), the 3% bonds were exchanged at 79 pesos for the nominal value of 100 pesos. Because the terms of the conversion were favorable, the majority of existing bondholders took the offer. In constructing our bond series for Argentina, we assume conversion at the 79:100 rate. We compute the price change and multiply by 0.79 to reflect the conversion when computing the capital gain for August 1960, and we add one month of coupon income at the 8% coupon rate to calculate the return.

A.4.6 Bond default in Greece

The bond return calculation must be adjusted in the event of a default or bond exchange that produces a change in par value. Defaults on domestic sovereign bonds are rare relative to external defaults, particularly for developed countries [Reinhart and Rogoff (2011)]. Rather, inflation is a more commonly used tool for eroding the real value of debt.

A notable event that produced a change in par value is the Greek default in 2012. Greece undertook a debt exchange in March 2012 in which creditors exchanged their existing bonds for a package of new government securities with a lower face value. Zettelmeyer, Trebesch, and Gulati (2013) provide an issue-by-issue estimate of the haircut for existing bondholders. We use the 53.8% haircut estimate for the bond with maturity closest to ten years. The ten-year bond yield declined substantially from 36.6% to 21.0% in March 2012, such that our calculation based on bond yields produces a nominal net return of 67.1%. Our calculation of the nominal gross return that incorporates the haircut is $1.671 \times (1-0.538) = 0.772$ to produce a nominal net return of -22.8% for ten-year bonds in March 2012.

A.4.7 Germany in 1919 to 1924 and 1948

To maintain consistency with our treatment of stock returns in Germany in the inflationary period from 1917 to 1923, we also compute bond returns in gold marks. We use bond prices in paper marks from Fischer (1923, 1924, 1925) and convert paper mark prices to gold marks by using the USD exchange rate because the United States was on the gold standard during that period. The change in gold mark bond prices provides an estimate of the capital appreciation of the bonds. We compute the total bond return by including interest payments based on the 3% coupon rate of the bonds. We use this approach from February 1919 to January 1924.

Germany exchanged Reichsmarks for Deutschemarks in June 1948. For government bonds, the exchange was 10:1 [Schnabl (2019)]. To reflect the economic value of the currency exchange, we adjust the bond price at the end of June 1948 by dividing the price of the bond by ten. The resulting nominal bond return in June 1948 is −90.0%.

A.5 Bills

We estimate returns on bills using short-term yields and rates. For most countries and periods, the GFDatabase has coverage with yield data on short-term (typically three-month) government bills. When these data are missing, we next use central bank rates when available and then interbank rates from the GFDatabase. We supplement these data with hand-collected, short-term rates from original source documents to achieve full coverage. We convert annual nominal rates on bills into monthly nominal returns denoted by $R_{i,t}^{Nominal\ bills}$ and then calculate real returns,

$$R_{i,t}^{Bills} = \frac{R_{i,t}^{Nominal\ bills}}{\Pi_{i,t}}. \tag{A9}$$

We compute monthly nominal bill returns from annual yields or rates as

$$R_{i,t}^{Nominal\ bills} = (1 + R_{i,t-1}^{Annual\ rate})^{1/12}, \tag{A10}$$

where $R_{i,t-1}^{Annual\ rate}$ is the annualized short-term government bill yield, central bank rate, or interbank rate reported at the end of month $t-1$.

A.5.1 Data issues related to bills

We have a few periods over which there are no bill data from GFD or alternative sources, and we are required to make assumptions to fill these gaps in the data. For Canada, we use a yield of 5.75% for the seven-month period from January 1914 to July 1914. This value is an average of the 6.50% interbank rate for December 1913 from GFD and the 5.00% advance rate for August 1914 from Shearer and Clark (1984). The Netherlands is missing data for February 2014, so we average the short-term government bill yields from GFD of 0.09% for January 2014 and 0.13% for March 2014.

For New Zealand from January 1896 to December 1914, we use short-term yields on bills held by the Post Office Savings Bank Fund. The Post Office Savings Bank Fund did not hold Treasury bills in 1913, so we are missing data for that year. The yields are 3.00% in December 1912 and 4.00% in January 1914, and we use the average of 3.50% to fill in the data gap. We are also missing yield data for New Zealand from January 1915 to December 1919. The yields for December 1914 and January 1920 are both 4.00%, so we assume a 4.00% yield over the adjoining period with missing data.

A.6 Other variables

We follow the data adjustments noted in Anarkulova, Cederburg, and O'Doherty (2022) and the corresponding internet appendix to estimate country-level inflation and exchange rate changes.

The data for market capitalization are from GFD. These series are typically reported at an annual frequency. There are missing data for some country-year observations of these series. For market capitalization, which is reported in USD, we fill data gaps by interpolating changes in proportion to USD nominal stock index returns. We use market capitalization series for Germany from 1917 to 1923 that are denominated in gold marks rather than paper marks. This approach is consistent with the calculation of the total return index for domestic stocks for Germany over this period. We fill a data gap in the 2023 GFD market capitalization data for the UK using an alternative source.[36]

Table A.V shows dividend-price ratio data for each country in the sample. We use annual dividend-price ratio data. We rely on external sources to calculate dividend-price ratios for Slovakia and Latvia. In both cases, GFD lacks comprehensive information to compute these ratios. For Slovakia, we use dividend-price ratio data from the Bratislava Stock Exchange's official website.[37] For Latvia, we calculate dividend-price ratios using data on total dividends paid by companies from Nasdaq Baltic and data on total market capitalization from GFD.[38] We use dividend-price ratios from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) for Portugal from 1934 to 1987.

We follow Anarkulova, Cederburg, and O'Doherty (2022) to fill the data gaps in GFD dividend-price ratio data for Chile from January 1967 to December 1970 and Czechoslovakia from April 1938 to March 1943. For Chile, we fill the data gap with a 7.0% yield based on the dividend yield observation in December 1966. The dividend yield in Czechoslovakia fluctuates between 1.4% and 2.6% in the three years before the break in the data, so we assume a 2.0% dividend yield for the missing observations. Table A.VI shows additional periods over which annual dividend-price ratio data are missing in the GFDatabase. We estimate dividend-price ratios for these periods with missing data using the methods described in the table.

A.7 External validation tests

This section details the external validation tests for our stock and bond return data.

A.7.1 Comparison of stock data from GFD and Jordà et al. (2019)

Anarkulova, Cederburg, and O'Doherty (2022) compare their data on stock returns from GFD with the stock returns from the overlapping periods in Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019). They find that the country-level samples from these two sources have very similar average returns, standard deviations, and extreme returns. They also show that the return correlation across the two sources exceeds 0.90 for nearly all countries. Given that our approach to constructing country-level stock returns closely follows the approach in Anarkulova, Cederburg, and O'Doherty (2022), these tests also provide external validation of our stock data.

A.7.2 Comparison of bond data from GFD and Datastream

As described in Section A.4, we calculate bond returns using bond yield data from GFD and other sources. In this section, we perform an external validation exercise by comparing our bond returns with those from Datastream over the periods and countries for which they are available. The analysis serves both to ensure that our approach to converting bond yields to returns is empirically accurate and to assess whether our bond return data and the bond return data from a popular alternative source exhibit common characteristics.

[36]The data are available at https://www.ceicdata.com/en/indicator/united-kingdom/market-capitalization.

[37]See http://www.bsse.sk/%C5%A0tatistika/Mesa%C4%8Dn%C3%A1.aspx.

[38]The dividend data for Latvia are available at https://nasdaqbaltic.com/statistics/en/statistics.

Table A.VII shows results from the external validation analysis. The table reports statistics for real returns. Our sample overlaps with Datastream for 27 countries. Datastream data begin in 1989 for several countries and more recently for others. The table reports the sample size, the arithmetic and geometric means, standard deviation, and minimum and maximum returns for our data, the corresponding statistics for Datastream data, and the correlation between our returns and those from Datastream.

Table A.VII indicates a close correspondence between our bond return data and those from Datastream. For nearly all countries, the means, standard deviations, and extreme returns are highly similar across the two data sources. The return correlations are above 0.90 for 24 of the 27 countries. Only Hungary, Mexico, and Singapore have correlations below 0.90. Of the 24 countries with high correlations, Greece is unique in Table A.VII as the only country with economically significant differences in the remaining summary statistics. We proceed to discuss these four exceptions.

Hungary and Singapore appear to be the simplest cases. We examine bond yields and returns across the two datasets. The GFDatabase and Datastream bond yields differ, sometimes substantially, for these two countries. To reconcile the differences, we collect ten-year historical bond yield data from the Magyar Nemzeti Bank (the central bank of Hungary) and the Monetary Authority of Singapore.[39] For Hungary, the correlation in yield changes from the central bank data and our data is near one, whereas the correlation between yield changes from the central bank and Datastream is much lower. For Singapore, the GFDatabase and Singaporean government data exactly match. The large deviations between Datastream and these other sources primarily occur in the first seven months of the sample, and the reported returns in Datastream imply changes in yields that are not reflected in the data from the Monetary Authority. Excluding the first seven months, the correlation between returns in our data and Datastream is 0.98. Our data appear reliable for these countries.

The bond yields for Mexico in our data and in Datastream are relatively similar. For several months in the sample, the reported Datastream return seems inconsistent with the reported yield change. For example, the reported yield increases by 0.08% in June 2015, but the reported return is 8.12%. We compare our calculated returns and the reported Datastream returns with the returns on the S&P/BMV Mexico Sovereign Bond Index in these months.[40] The S&P/BMV index tracks bonds with several maturities, and its duration is low compared with the other two series. Nonetheless, the returns from this index are much more consistent with our data versus Datastream. In June 2015, for example, the S&P/BMV index reports a return of −0.15%, which is close to our return calculation of −0.10% but far from the 8.12% reported return in Datastream. Given the consistency between the GFDatabase and the S&P/BMV index, the deviations between our data and Datastream appear to be reporting errors for returns in Datastream.

The largest deviations in bond returns for Greece are related to the Greek bond default in 2012. As discussed in Section A.4.6, we calculate a bond return in March 2012 that accounts for the bond exchange and the associated haircut. Our return calculation, which reflects information from ten-year bond yields and the default, is −22.80% in this month. This return differs substantially from the −4.16% return reported by Datastream. Our study focuses on domestic debt, so we take the perspective of a hypothetical domestic investor. Participation rates in the exchange were higher among domestic investors compared with international investors [Zettelmeyer, Trebesch, and Gulati (2013)]. We do not have information on Datastream's return calculation for this month, but the difference could arise from a different assumption about participation in the exchange. Late in 2012, Greece announced a voluntary bond buyback to be executed in December 2012, and the buyback led to an increase in market prices [Zettelmeyer, Trebesch, and Gulati (2013)]. We observe a 4.32% decrease in bond yield in December

[39] See https://www.mnb.hu/en/statistics/statistical-data-and-information/statistical-time-series/xi-money-and-capital-markets and https://eservices.mas.gov.sg/statistics/fdanet/BenchmarkPricesAndYields.aspx.

[40] See https://www.spglobal.com/spdji/en/indices/fixed-income/sp-bmv-mexico-sovereign-bond-index/.

2012 and calculate a return of 26.12%. Datastream reports a 3.21% decrease in bond yield and reports a return of 41.47%, such that the return is much larger than that implied by the yield change. Given that the buyback occurred at prevailing market prices, our view is that any effect of the buyback should be reflected in the change in yields. The return differences for these two months account for much of the difference in average returns for Greece in Table A.VII.

Table A.I: Developed country sample periods

The table shows developed countries, initial development dates, classification reasons for development, sample eligibility details, and sample coverage. The development year classifications are based on agricultural labor share or organizational membership in the Organisation for European Economic Co-operation (OEEC) or the Organisation for Economic Co-operation and Development (OECD). Sample eligibility for a given developed country requires that the country has issued long-term government bonds. The sample period start date is the later of the sample eligibility date and the first date with return data for stocks, bonds, and bills.

	Development details		Sample eligibility details		Sample coverage		
Country	Year	Reason for classification	Year	Reason for delayed sample eligibility	Start date	End date	Coverage (%)
United Kingdom	1841	Agricultural labor share	1890	Sample for study starts in 1890	1890:01	2023:12	100.0
Netherlands	1849	Agricultural labor share	1890	Sample for study starts in 1890	1914:01	2023:12	82.1
Belgium	1856	Agricultural labor share	1890	Sample for study starts in 1890	1897:01	2023:12	94.8
France	1866	Agricultural labor share	1890	Sample for study starts in 1890	1890:01	2023:12	100.0
Norway	1875	Agricultural labor share	1890	Sample for study starts in 1890	1914:01	2023:12	82.1
Germany	1882	Agricultural labor share	1890	Sample for study starts in 1890	1890:01	2023:12	100.0
Denmark	1890	Agricultural labor share	1890	n/a	1890:01	2023:12	100.0
Switzerland	1890	Agricultural labor share	1890	n/a	1914:01	2023:12	82.1
United States	1890	Agricultural labor share	1890	n/a	1890:01	2023:12	100.0
Canada	1891	Agricultural labor share	1891	n/a	1891:01	2023:12	100.0
Argentina	1895	Agricultural labor share	1895	n/a	1947:02	1966:12	27.7
New Zealand	1896	Agricultural labor share	1896	n/a	1896:01	2023:12	100.0
Australia	1901	Agricultural labor share	1901	n/a	1901:01	2023:12	100.0
Sweden	1910	Agricultural labor share	1910	n/a	1910:01	2023:12	100.0
Austria	1920	Agricultural labor share	1920	n/a	1920:01	2023:12	100.0
Chile period I	1920	Agricultural labor share	1920	n/a	1927:01	1970:12	86.3
Greece	1920	Agricultural labor share	1920	n/a	1981:02	2023:12	41.3
Czechoslovakia	1921	Agricultural labor share	1921	n/a	1922:05	1945:05	94.5
Japan	1930	Agricultural labor share	1930	n/a	1930:01	2023:12	100.0
Portugal	1930	Agricultural labor share	1930	n/a	1934:01	2023:12	95.7
Italy	1931	Agricultural labor share	1931	n/a	1931:01	2023:12	100.0
Ireland	1936	Agricultural labor share	1936	n/a	1936:01	2023:12	100.0
Singapore	1947	Agricultural labor share	1998	Long-term bonds first available in 1998	1998:07	2023:12	100.0
Iceland	1948	OEEC membership	1992	Long-term bonds first available in 1992	2002:01	2023:12	68.8
Luxembourg	1948	OEEC membership	1948	n/a	1982:01	2023:12	55.3
Türkiye	1948	OEEC membership	2010	Long-term bonds first available in 2010	2010:02	2023:12	100.0

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Table A.I (Continued)

	Development details		Sample eligibility details		Sample coverage		
Country	Year	Reason for classification	Year	Reason for delayed sample eligibility	Start date	End date	Coverage (%)
Spain	1959	OEEC membership	1959	n/a	1959:01	2023:12	100.0
Finland	1969	OECD membership	1969	n/a	1969:01	2023:12	100.0
Mexico	1994	OECD membership	2001	Long-term bonds first available in 2001	2001:08	2023:12	100.0
Czech Republic	1995	OECD membership	2000	Long-term bonds first available in 2000	2000:05	2023:12	100.0
Hungary	1996	OECD membership	1999	Long-term bonds first available in 1999	1999:02	2023:12	100.0
Poland	1996	OECD membership	1999	Long-term bonds first available in 1999	1999:06	2023:12	100.0
South Korea	1996	OECD membership	2000	Long-term bonds first available in 2000	2000:11	2023:12	100.0
Slovakia	2000	OECD membership	2000	n/a	2000:01	2023:12	100.0
Chile period II	2010	OECD membership	2010	n/a	2010:01	2023:12	100.0
Estonia	2010	OECD membership	—	No qualifying long-term bonds	—	—	—
Israel	2010	OECD membership	2010	n/a	2010:01	2023:12	100.0
Slovenia	2010	OECD membership	2010	n/a	2010:01	2023:12	100.0
Latvia	2016	OECD membership	2016	n/a	2016:01	2023:12	100.0
Lithuania	2018	OECD membership	2018	n/a	2018:01	2023:12	100.0
Colombia	2020	OECD membership	2020	n/a	2020:01	2023:12	100.0

Table A.II: Data sources

The table summarizes the data series used to compute returns for each developed country in the sample. For each country and asset class, the table reports the data series used to construct returns and the corresponding start and end dates. The data series symbols correspond to those in the GFDatabase from Global Financial Data. For stocks and bills, the table also indicates the data series type. Stock returns are based on either total return indexes (TRI) or combinations of price indexes and dividend-price ratios (PI/DP). Bill returns are based on short-term Treasury yields (TBY), central bank interest rates (CBR), interbank interest rates (IIR), one-year government bond yields (GBY-1), deposit interest rates (DIR), interest rates on advances (IRA), overnight interest rates (OIR), or time money rates (TMR). All bond returns correspond to returns on ten-year government bonds. We provide details on alternative data sources in the footnotes.

	Stocks				Bonds			Bills			
Country	Series	Type	Start	End	Series	Start	End	Series	Type	Start	End
Argentina	_IBGD, SYARGYM	PI/DP	1947:02	1966:12	❸	1947:02	1966:12	IDARGD	CBR	1947:02	1966:12
Australia	_AORDAD	TRI	1901:01	2023:12	IGAUS10D	1901:01	2023:12	❷	TBY	1901:01	1920:06
								IDAUSD	CBR	1920:07	1928:06
								ITAUS3D	TBY	1928:07	2023:12
Austria	_ATXTRD	TRI	1920:01	2023:12	IGAUT10D	1920:01	2023:12	IDAUTD	CBR	1920:01	1959:12
								ITAUT3M	TBY	1960:01	1990:12
								IGAUT1D	GBY-1	1991:01	2023:12
Belgium	_BCSHD	TRI	1897:01	2023:12	IGBEL10D	1897:01	2023:12	IDBELD	CBR	1897:01	1947:12
								ITBEL3D	TBY	1948:01	2023:12
Canada	_TRGSPTSE	TRI	1891:01	2023:12	IGCAN10D	1891:01	2023:12	✚	DIR	1891:01	1901:12
								IMCANMOM	OIR	1902:01	1913:12
								✲	IRA	1914:08	1934:02
								ITCAN3D	TBY	1934:03	2023:12
Chile period I	_IGPAD, SYCHLYM	PI/DP	1927:01	1970:12	❹	1927:01	1929:09	IDCHLD	CBR	1927:01	1970:12
					❺	1929:10	1930:12				
					IGCHLCM	1931:01	1956:02				
					❻	1956:03	1970:12				
Chile period II	_IPSAD	TRI	2010:01	2023:12	IGCHLIM	2010:01	2015:03	ITCHL3D	TBY	2010:01	2012:09
					❼	2015:04	2023:12	IDCHLD	CBR	2012:10	2023:12
Colombia	TRCOLSTM	TRI	2020:01	2023:12	IGCOL10D	2020:01	2023:12	ITCOL3W	TBY	2020:01	2023:12
Czech Republic	_PXTRD	TRI	2000:05	2023:12	IGCZE10D	2000:05	2023:12	ITCZE3D	TBY	2000:05	2017:02
								IDCZED	CBR	2017:03	2023:12
Czechoslovakia	CZINDXM, SYCZEYM	PI/DP	1922:05	1937:11	❹	1922:05	1927:01	IDCZED	CBR	1922:05	1945:05
	CZINDEXM, SYCZEYM	PI/DP	1937:12	1943:03	❺	1927:02	1944:06				

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Table A.II (Continued)

	Stocks				Bonds			Bills			
Country	Series	Type	Start	End	Series	Start	End	Series	Type	Start	End
Denmark	_OMXCGID	TRI	1890:01	2023:12	IGDNK10D	1890:01	2023:12	IDDNKD	CBR	1890:01	1975:12
								ITDNK3D	TBY	1976:01	2023:12
Finland	_OMXHGID	TRI	1969:01	2023:12	IGFIN10D	1969:01	2023:12	IDFIND	CBR	1969:01	1996:08
								IGFIN1D	GBY-1	1996:09	2012:01
								ITFIN1D	TBY	2012:02	2013:05
								IGFIN1D	GBY-1	2013:06	2014:07
								IDFIND	CBR	2014:08	2023:12
France	TRSBF250D	TRI	1890:01	2023:12	IGFRA10D	1890:01	2023:12	IDFRAD	CBR	1890:01	1930:12
								ITFRA3D	TBY	1931:01	2023:12
Germany	_CDAXD	TRI	1890:01	2023:12	IGDEU10D	1890:01	2023:12	IDDEUD	CBR	1890:01	1952:12
								ITDEU3D	TBY	1953:01	2023:12
Greece	_RETMD	TRI	1981:02	2023:12	IGGRC10D	1981:02	2014:01	ITGRC3D	TBY	1981:02	2023:12
					❼	2014:02	2014:02				
					IGGRC10D	2014:03	2023:12				
Hungary	_BUXD	TRI	1999:02	2023:12	IGHUN10D	1999:02	2023:12	ITHUN3D	TBY	1999:02	2023:12
Iceland	_OMXIPID, SYISLYM	PI/DP	2002:01	2002:06	❼	2002:01	2004:02	ITISL3D	TBY	2002:01	2013:01
	_OMXIGID	TRI	2002:07	2023:12	IGISL10D	2004:03	2023:12	IDISLD	CBR	2013:02	2023:12
Ireland	_IVRTD	TRI	1936:01	2023:12	IGIRL10D	1936:01	2023:12	IDIRLD	CBR	1936:01	1969:11
								ITIRL3M	TBY	1969:12	2008:12
								❼	IIR	2009:01	2023:12
Israel	TRISRSTM	TRI	2010:01	2023:12	❼	2010:01	2014:12	ITISR3D	TBY	2010:01	2023:12
					IGISR10IM	2015:01	2023:12				
Italy	_BCIPRD	TRI	1931:01	2019:12	IGITA10D	1931:01	2023:12	IDITAD	CBR	1931:01	1939:12
	_FTITLMS, SYITAYM	PI/DP	2020:01	2023:12				ITITA3D	TBY	1940:01	2023:12
Japan	_TOPXDVD	TRI	1930:01	2023:12	IGJPN10D	1930:01	2023:12	IDJPND	CBR	1930:01	1959:12
								ITJPN3D	TBY	1960:01	2023:12
Latvia	_OMXRGID	TRI	2016:01	2023:12	❼	2016:01	2023:12	❼	IIR	2016:01	2023:12
Lithuania	_OMXVGID	TRI	2018:01	2023:12	IGLTU10D	2018:01	2023:01	❼	IIR	2018:01	2023:12
					❼	2023:02	2023:12				
Luxembourg	_LUXXD, SYLUXYM	PI/DP	1982:01	1984:12	IGLUX10D	1982:01	2023:03	IMLUXM	OIR	1982:01	1998:12
	_LUXXRD	TRI	1985:01	2016:11	❼	2023:04	2023:12	❼	IIR	1999:01	2023:12
	❶	TRI	2016:12	2023:12							
Mexico	_IRTD	TRI	2001:08	2023:12	IGMEX10D	2001:08	2023:12	ITMEX3D	TBY	2001:08	2023:12

(Continued on next page)

Table A.II (Continued)

	Stocks				Bonds			Bills			
Country	Series	Type	Start	End	Series	Start	End	Series	Type	Start	End
Netherlands	_AAXGRD	TRI	1914:01	2023:12	IGNLD10D	1914:01	1944:08	IDNLDBD	CBR	1914:01	1940:12
					❷	1944:09	1945:12	ITNLD3D	TBY	1941:01	2023:12
					IGNLD10D	1946:01	2023:12				
New Zealand	_NZGID	TRI	1896:01	2023:12	IGNZL10D	1896:01	2023:12	■	TBY	1896:01	1919:12
								■	DIR	1920:01	1922:12
								IDNZLD	CBR	1923:01	1978:02
								ITNZL3D	TBY	1978:03	2023:12
Norway	_OSEAXD	TRI	1914:01	2023:12	IGNOR10D	1914:01	2023:12	IDNORD	CBR	1914:01	1941:11
								ITNOR3D	TBY	1941:12	2023:12
Poland	_WIGD	TRI	1999:06	2023:12	IGPOL10D	1999:06	2023:03	ITPOL3D	TBY	1999:06	2023:12
					❼	2023:04	2023:12				
Portugal	_IBTAD, ❷	PI/DP	1934:01	1988:01	IGPRT10D	1934:01	1974:04	IDPRTD	CBR	1934:01	1988:12
	_BVLGD	TRI	1988:02	2023:12	❷	1974:05	1975:12	ITPRT6D	TBY	1989:01	1999:01
					IGPRT10D	1976:01	2023:06	IDPRTD	CBR	1999:02	2001:12
					❼	2023:07	2023:12	❷	TBY	2002:01	2010:09
								ITPRT6D	TBY	2010:10	2022:09
								❼	IIR	2022:10	2023:12
Singapore	_TFTFSTD	TRI	1998:07	2023:12	IGSGP10D	1998:07	2023:12	ITSGP3D	TBY	1998:07	2023:12
Slovakia	_SAXD	TRI	2000:01	2023:12	IGSVK10D	2000:01	2023:03	IDSVKD	CBR	2000:01	2008:12
					❼	2023:04	2023:12	❼	IIR	2009:01	2023:12
Slovenia	_SBITOPD, SYSVNYM	PI/DP	2010:01	2023:12	❼	2010:01	2023:12	ITSVN3M	TBY	2010:01	2023:12
South Korea	TRKORSTM	TRI	2000:11	2023:12	IGKOR10D	2000:11	2023:12	IGKOR1D	GBY-1	2000:11	2023:12
Spain	_BCNPR30	TRI	1959:01	2023:12	IGESP10D	1959:01	2023:12	IDESPD	CBR	1959:01	1978:12
								ITESP12D	TBY	1979:01	2023:12
Sweden	_OMXSBGI	TRI	1910:01	2023:12	IGSWE10D	1910:01	2018:12	IDSWED	CBR	1910:01	1954:12
					❼	2019:01	2019:01	ITSWE3D	TBY	1955:01	2023:12
					IGSWE10D	2019:02	2023:12				
Switzerland	_SSHID	TRI	1914:01	2023:12	IGCHE10D	1914:01	1941:12	IDCHED	CBR	1914:01	1979:12
					❽	1942:01	1942:01	ITCHE3D	TBY	1980:01	2023:12
					IGCHE10D	1942:02	2023:12				
Türkiye	TRRBILED	TRI	2010:02	2023:12	IGTUR10D	2010:02	2023:06	ITTUR3D	TBY	2010:02	2014:09
					❾	2023:07	2023:12	IGTUR1D	GBY-1	2014:10	2023:03
								❿	TBY	2023:04	2023:12

(Continued on next page)

Table A.II (Continued)

	Stocks				Bonds			Bills			
Country	Series	Type	Start	End	Series	Start	End	Series	Type	Start	End
United Kingdom	_TFTASD	TRI	1890:01	2023:12	IGGBR10D	1890:01	2023:12	IDGBRD	CBR	1890:01	1899:12
								ITGBR3D	TBY	1900:01	2023:12
United States	_SPXTRD	TRI	1890:01	2023:12	IGUSA10D	1890:01	2023:12	▲	TMR	1890:01	1914:10
								IDUSAD	CBR	1914:11	1919:12
								ITUSA3CMD	TBY	1920:01	2023:12

Footnotes:

❶ Stock returns for Luxembourg for the period from 2016:12 to 2023:12 are from the Luxembourg Stock Exchange. See https://www.bourse.lu/home.

❷ Dividend-price ratios, bond returns, and bill returns for several countries are from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019).

❸ Bond returns for Argentina for the period from 1947:02 to 1966:12 are based on data from the Central Bank of Argentina. See http://www.bcra.gov.ar/PublicacionesEstadisticas/Boletin_estadistico.asp.

❹ Bond returns for select periods in Chile and Czechoslovakia are based on London quotes from the International Center for Finance at Yale. See https://som.yale.edu/faculty-research/our-centers-initiatives/international-center-finance/data/historical-financial-research-data/london-stock-exchange.

❺ Bond returns for select periods in Chile and Czechoslovakia are based on London quotes from the League of Nations reports.

❻ Bond returns for Chile for the period from 1956:03 to 1970:12 are based on data from the Central Bank of Chile. See https://repositoriodigital.bcentral.cl/xmlui/handle/20.500.12580/26/browse?type=dateissued.

❼ Bond and bill returns for several countries are based on data from Federal Reserve Economic Data (FRED) at the Federal Reserve Bank of St. Louis. See https://fred.stlouisfed.org/.

❽ The bond return in Switzerland for 1942:01 is based on data from the Swiss National Bank (https://www.snb.ch/de/iabout/stat/statrep/statpubdis/id/statpub_histz_arch#t2). We also shift a portion of the series IGCHE10D from GFD, which originally covers the period from 1942:01 to 1990:11, to cover the period from 1942:02 to 1990:12.

❾ Bond returns in Türkiye for the period from 2023:07 to 2023:12 are based on data from http://www.worldgovernmentbonds.com/bond-historical-data/turkey/10-years/.

❿ Bill returns for Türkiye for the period from 2023:04 to 2023:12 are based on data from https://www.worldgovernmentbonds.com/bond-historical-data/turkey/3-months/.

✚ Bill returns for Canada for the period from 1891:01 to 1901:12 are based on interest rates on deposits in government savings banks. See p. 363 of https://www66.statcan.gc.ca/eng/1901-eng.htm.

✲ Bill returns for Canada for the period from 1914:08 to 1934:02 are based on data from Shearer and Clark (1984).

■ Bill returns for New Zealand for the period from 1896:01 to 1922:12 are based on data from the annual New Zealand Official Year-book. See, e.g., https://www3.stats.govt.nz/New_Zealand_Official_Yearbooks/1896/NZOYB_1896.html.

▲ Bill returns for the United States for the period from 1890:01 to 1914:10 are based on data from Macaulay (1938).

Table A.III: Multi-month stock returns

The table reports periods of multi-month stock returns associated with exchange closures and details our approach to converting each return to a series of monthly returns. For each multi-month return observation, the table reports the number of months, the start and end dates of the period, the nominal and real net stock market returns earned over the period, and the adjustment method. For adjustment method 1, we use alternative data sources from GFD (e.g., black market trading data) to fill in a complete series of monthly returns. For adjustment method 2, we assign the full multi-month real return to the first month of the period and assign zero real returns to the remaining months. The cases marked with a ✚ are discussed in Section A.2.1. Panels A and B show events corresponding to World War I and World War II, respectively, Panel C shows periods with revolutions, Panel D shows financial and banking crises, and Panel E shows labor strikes.

Country	Months	Start date	End date	Nominal return (%)	Real return (%)	Adjustment
Panel A: World War I						
Australia	6	1914:08	1915:01	−0.45	−0.39	Method 1
Belgium	52	1914:08	1918:11	25.12	−55.91	Method 2
Canada	7	1914:08	1915:02	1.38	−3.59	Method 1
Denmark	4	1914:08	1914:11	0.72	−0.27	Method 1
France	6	1914:08	1915:01	−10.89	−27.54	Method 1
Germany	42	1914:08	1918:01	20.03	−38.87	Method 1
Netherlands	7	1914:08	1915:02	−1.23	−3.50	Method 1
Norway	3	1914:08	1914:10	−3.80	−4.36	Method 2
Sweden	4	1914:08	1914:11	−5.91	−8.96	Method 2
Switzerland	24	1914:08	1916:07	0.17	−18.71	✚
United Kingdom	6	1914:08	1915:01	0.11	−2.94	Method 1
United States	5	1914:08	1914:12	−2.14	−3.11	Method 1
Panel B: World War II						
Austria	2	1938:04	1938:05	6.01	5.64	Method 2
Austria	113	1939:07	1948:11	315.61	−16.90	✚
Belgium	5	1940:06	1940:10	22.38	12.54	Method 2
Belgium	11	1944:08	1945:06	−0.29	−17.08	Method 2
Czechoslovakia	16	1938:10	1940:01	32.23	16.91	Method 2
Czechoslovakia	4	1942:01	1942:04	20.66	12.32	Method 2
Denmark	2	1940:05	1940:06	−7.64	−10.67	Method 2
France	2	1939:09	1939:10	−2.96	0.53	Method 1
France	10	1940:06	1941:03	94.57	75.61	Method 2
Germany	67	1943:01	1948:07	−87.62	−91.10	Method 2
Japan	45	1945:09	1949:05	449.38	−87.15	Method 1
Netherlands	5	1940:05	1940:09	20.63	15.21	Method 2
Netherlands	21	1944:09	1946:05	−14.33	−33.15	Method 2
Norway	2	1940:04	1940:05	−2.07	−3.52	Method 1
Switzerland	2	1940:06	1940:07	−3.57	−5.11	Method 1
Panel C: Revolution						
Czechoslovakia	26	1943:04	1945:05	−90.00	−88.59	✚
Portugal	35	1974:05	1977:03	−80.40	−89.24	Method 2

(Continued on next page)

Table A.III (Continued)

Country	Months	Start date	End date	Nominal return (%)	Real return (%)	Adjustment
Panel D: Financial or banking crisis						
Austria	2	1931:10	1931:11	6.25	5.60	Method 2
Germany	2	1931:08	1931:09	−24.58	−23.01	Method 2
Germany	7	1931:10	1932:04	−8.22	1.78	Method 2
Greece	2	2015:07	2015:08	−21.53	−20.13	Method 2
Panel E: Labor strike						
France	2	1974:04	1974:05	−6.17	−8.76	Method 1
France	2	1979:03	1979:04	12.79	10.69	Method 1

Table A.IV: Bond return smoothing

The table summarizes periods over which we are missing bond yield data. In each case, we use the country-level yield data from before and after the missing observations to produce a series of constant monthly returns across the period noted in the table. For each period with missing bond data, the table reports the country, the number of missing observations, and the start and end dates of the period.

Country	Months	Start date	End date
Argentina	4	1948:08	1948:11
	11	1949:01	1949:11
	11	1950:01	1950:11
	11	1951:01	1951:11
	11	1952:01	1952:11
	11	1953:01	1953:11
	11	1954:01	1954:11
	24	1955:01	1956:12
	1	1958:02	1958:02
	1	1958:08	1958:08
	1	1959:05	1959:05
	1	1959:08	1959:08
Belgium	3	1940:05	1940:07
Czechoslovakia	15	1938:10	1939:12
Finland	1	1991:06	1991:06
Germany	8	1931:08	1932:03
	25	1943:12	1945:12
Greece	44	1989:01	1992:08
Netherlands	2	1940:05	1940:06
	3	1944:09	1944:11
	11	1945:01	1945:11
Portugal	7	1974:05	1974:11
	11	1975:01	1975:11
	1	2014:02	2014:02
Switzerland	5	1914:08	1914:12

Table A.V: Dividend-price ratio data

The table shows dividend-price ratio data for each country in the sample. The data are annual, and the data series symbols correspond to those in the GFDatabase from Global Financial Data. We provide details on alternative sources in Section A.6.

Country	Series	Start year	End year
Argentina	SYARGYM	1947	1966
Australia	SYAUSYM	1901	2023
Austria	SYAUTYM	1920	2023
Belgium	SYBELYM	1897	2023
Canada	SYCANYTM	1891	2023
Chile period I	SYCHLYM	1927	1970
Chile period II	SYCHLYM	2010	2023
Colombia	SYCOLYM	2020	2023
Czech Republic	SYCZEYM	2000	2023
Czechoslovakia	SYCZEYM	1922	1945
Denmark	SYDNKYM	1890	2023
Finland	SYFINYM	1969	2023
France	SYFRAYM	1890	2023
Germany	SYDEUYM	1890	2023
Greece	SYGRCYM	1981	2023
Hungary	SYHUNYM	1999	2023
Iceland	SYISLYM	2002	2023
Ireland	SYIRLYM	1936	2023
Israel	SYISRYM	2010	2023
Italy	SYITAYM	1931	2023
Japan	SYJPNYM	1930	2023
Latvia	See table caption	2016	2023
Lithuania	SYLTUYM	2018	2023
Luxembourg	SYLUXYM	1982	2023
Mexico	SYMEXYM	2001	2023
Netherlands	SYNLDYAM	1914	2023
New Zealand	SYNZLYM	1896	2023
Norway	SYNORYM	1914	2023
Poland	SYPOLYM	1999	2023
Portugal	See table caption	1934	1987
	SYPRTYM	1988	2023
Singapore	SYSGPYM	1998	2023
Slovakia	See table caption	2000	2023
Slovenia	SYSVNYM	2010	2023
South Korea	SYKORYM	2000	2023
Spain	SYESPYM	1959	2023
Sweden	SYSWEYM	1910	2023
Switzerland	SYCHEYM	1914	2023
Türkiye	SYTURYM	2010	2023
United Kingdom	_DFTASD	1890	2023
United States	SYUSAYM	1890	2023

Table A.VI: Data gaps for dividend-price ratios

The table shows periods over which we are missing annual dividend-price ratio data. For each period, the table reports the country, the number of missing annual observations, the start and end dates for the period, and the method used to estimate the missing ratios. For estimation method 1, we infer annual dividend-price ratios using total return index and price index data. For estimation method 2, we fill in missing dividend-price ratios using the last non-missing dividend-price ratio. For estimation method 3, we use the next year's dividend-price ratio.

Country	Years	Start year	End year	Method
Denmark	31	1938	1968	Method 1
France	1	1940	1940	Method 2
Germany	7	1944	1950	Method 2
Iceland	1	2001	2001	Method 3
	17	2007	2023	Method 1
Italy	1	1945	1945	Method 1
Japan	4	1945	1948	Method 2
Luxembourg	1	1981	1981	Method 3
	29	1995	2023	Method 1
Mexico	1	2014	2014	Method 1
Singapore	1	2021	2021	Method 1
Switzerland	4	1914	1917	Method 2

Table A.VII: External validation test results

The table reports summary statistics for monthly real net bond returns for each developed country with a return sample that overlaps with the sample from Datastream. For each country, the table shows the number of sample months. The table also shows the following summary statistics for our sample and for the Datastream sample: the arithmetic average return ($\bar{R}_a$), the geometric average return ($\bar{R}_g$), the standard deviation of return (SD), the minimum (Min) and the maximum (Max) return, and the correlation between the return samples (Corr).

		Our data					Datastream					
Country	Months	$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	Corr
Australia	420	0.39	0.36	2.19	−5.63	6.80	0.40	0.38	2.22	−6.75	7.84	0.97
Austria	420	0.21	0.19	1.85	−7.31	5.99	0.26	0.25	1.67	−7.10	4.99	0.96
Belgium	414	0.27	0.25	1.94	−6.49	7.47	0.33	0.31	1.92	−7.48	8.33	0.98
Canada	420	0.33	0.31	2.02	−6.33	6.47	0.34	0.32	1.99	−6.11	6.36	0.97
Czech Republic	284	0.11	0.08	2.45	−8.47	9.62	0.17	0.15	2.35	−8.49	8.76	0.94
Denmark	419	0.31	0.29	2.11	−7.05	6.40	0.34	0.32	2.00	−7.02	5.83	0.98
Finland	388	0.34	0.32	2.16	−7.16	8.59	0.38	0.36	1.96	−7.55	8.01	0.97
France	420	0.31	0.29	1.92	−7.15	5.86	0.35	0.34	1.81	−5.85	5.19	0.97
Germany	420	0.23	0.21	1.86	−6.84	5.31	0.26	0.24	1.78	−6.93	5.08	0.97
Greece	297	0.33	0.14	5.95	−30.84	26.45	0.54	0.28	7.08	−41.35	41.84	0.93
Hungary	299	0.23	0.17	3.49	−9.83	12.42	0.24	0.18	3.41	−16.31	15.34	0.86
Ireland	420	0.34	0.30	2.62	−14.79	15.45	0.35	0.32	2.42	−14.16	14.54	0.97
Italy	393	0.42	0.38	2.51	−8.03	10.26	0.44	0.41	2.38	−11.27	8.62	0.94
Japan	420	0.19	0.18	1.45	−7.76	5.30	0.24	0.23	1.33	−6.15	5.19	0.95
Mexico	162	0.13	0.09	2.60	−7.25	5.76	0.12	0.09	2.38	−7.18	7.95	0.87
Netherlands	420	0.22	0.20	1.93	−8.54	6.71	0.27	0.25	1.83	−8.22	5.21	0.97
New Zealand	393	0.38	0.36	2.10	−8.40	7.28	0.40	0.38	2.04	−7.11	6.93	0.95
Norway	373	0.29	0.27	1.96	−6.24	6.01	0.28	0.27	1.85	−6.48	5.50	0.97
Poland	276	0.32	0.28	2.85	−11.06	12.51	0.31	0.27	2.92	−11.71	13.92	0.94
Portugal	365	0.40	0.35	3.18	−13.23	14.98	0.45	0.40	3.16	−14.84	20.14	0.95
Singapore	180	−0.01	−0.03	1.89	−6.13	4.94	0.15	0.13	1.90	−4.66	11.07	0.88
South Korea	141	0.12	0.10	1.84	−4.75	6.05	0.14	0.12	1.72	−3.90	5.88	0.98
Spain	397	0.37	0.34	2.48	−9.92	9.47	0.42	0.40	2.38	−9.76	9.65	0.96
Sweden	420	0.35	0.32	2.30	−7.82	6.77	0.38	0.36	2.12	−8.48	5.62	0.98
Switzerland	420	0.19	0.18	1.65	−4.86	5.88	0.23	0.21	1.53	−4.43	5.06	0.94
United Kingdom	420	0.23	0.20	2.27	−10.62	8.23	0.24	0.22	2.16	−10.79	7.03	0.98
United States	420	0.24	0.22	2.23	−7.34	11.71	0.23	0.20	2.20	−6.24	11.69	0.98

B Social Security benefits calculations

We compute Social Security benefit payments using the formulas effective in 2022. Given that Social Security formulas are inflation-indexed, our calculations use real 2022 USD. As such, the amount subject to Social Security taxes for each year in the investor's working life is the minimum of annual income and $147,000, which is the maximum taxable earnings. The average indexed monthly earnings (AIME) for each investor is the average of their highest 35 years of taxed earnings divided by 12. An investor who retires at the normal retirement age of 67 has a personal Social Security benefit according to a formula that sums 90% of AIME up to $1,024, 32% of AIME between $1,024 and $6,172, and 15% of AIME in excess of $6,172. The retirement benefit reduces by (5/9)% per month of early retirement up to 36 months and further reduces by (5/12)% per month of early retirement between ages 62 (the earliest allowed retirement age) and 64 (normal retirement age minus 36 months). The retirement benefit increases by (2/3)% per month of late retirement between ages 67 and 70 (the latest allowed retirement age).

Spouses may be eligible for additional benefits under the Social Security system. Some investors may optimally choose to take spousal and/or survivor benefits. When both members of the couple are living, each spouse has the option to take half of their spouse's personal retirement benefit as a spousal benefit in lieu of taking their own retirement benefit. This option becomes useful if one of the two household members earns substantially more than the other during their working years. The spousal benefit is reduced by (25/36)% per month of early retirement up to 36 months and further reduced by (5/12)% per month between ages 62 and 64. Upon the death of one spouse, the surviving spouse qualifies for a survivor benefit. The potential survivor benefit is the personal benefit of the deceased spouse if the surviving spouse is full retirement age. If not, the survivor benefit is reduced by up to 28.5% (if taken before age 60). The surviving spouse may take the larger of their personal retirement benefit and their survivor benefit.

Finally, the SSA administers the Supplemental Security Income (SSI) program, which provides payments to retirees (and certain other individuals) who have little income from other sources. SSI payments are reduced by earnings, and the maximum monthly benefit amounts in 2022 are $1,261 for couples and $841 for singles. We impose minimum monthly consumption levels of $1,261 for couples and $841 for singles to reflect SSI payments. This modeling choice reflects the social safety net and avoids issues with computing utility when consumption levels are zero or low.

C Supplemental results

This appendix contains supplemental empirical results.

C.1 Household longevity

Table C.I summarizes the distribution of age at death in years conditional on survival to age 25 based on the SSA data and our simulation procedure described in Section 4. The table reports the mean, standard deviation, and distributional percentiles for age at death for the heterosexual couple, the female, the male, the female couple, and the male couple. The statistics for the couples correspond to the age of the last survivor from the couple at death. In the base case of a heterosexual couple, the mean age of the last survivor at death is 87.6 years, and the median age is 88.9 years. There is considerable uncertainty over longevity outcomes. The 5th percentile of age at death for the couple is 70.8 years, and the 95th percentile is 100.0 years. This uncertainty is an important feature to consider in assessing the ability of investment strategies to fund consumption through retirement. The last column of Table C.I reports the likelihood that a given investor type dies before reaching retirement age. There is a 19.5% (11.9%) chance that the male (female) dies before age 65, and there is a 2.3% chance that neither member of the heterosexual couple survives into the retirement period.

C.2 Conditional strategy performance

In Section 5.2.2, we show that the optimal age-based strategy outperforms the four benchmark strategies in preserving capital during the retirement period (i.e., the optimal strategy leads to the lowest probability of financial ruin under the 4% withdrawal rule). The outperformance of the optimal age-based strategy in capital preservation during retirement challenges the traditional view that investors should increase their allocation to bonds as they age. In this section, we further characterize this result by examining ruin probabilities conditional on couple and market outcomes. Figure C.1 plots conditional ruin probabilities for three benchmark strategies and the optimal strategy (bills, with their exceedingly high ruin probabilities, are omitted to enhance the readability of the figure). In each panel, we divide the 1,000,000 simulation draws into quintiles based on an outcome and plot the ruin probability within each quintile.

Panel A of Figure C.1 examines ruin probabilities in relation to couple longevity. Short-lived couples are unlikely to exhaust their savings, and the ruin probabilities range from 0.2% (optimal strategy) to 3.3% (domestic stocks). Differences across strategies are the most stark for long-lived couples. The optimal strategy produces a 14.9% probability of ruin, which is more than twice as high as the unconditional 6.7% probability. The other strategies fare much worse with high longevity, with ruin probabilities of 29.4% for domestic stocks, 32.2% for the balanced portfolio, and 41.2% for the TDF. These results emphasize that continuing to generate wealth throughout retirement is crucial when investors may have a relatively long retirement period. The poor unconditional performance of the TDF, which invests little in stocks during retirement, is partially attributable to its struggle to preserve capital for couples who live long lives.

Panel B conditions on the cumulative real domestic stock return in retirement. In the worst quintile of realized returns, the optimal age-based strategy has a ruin probability of 17.7%, which far exceeds the unconditional probability. But the optimal strategy is still the safest when domestic stocks do poorly. The domestic stock strategy is hardest hit, naturally, with a 52.4% probability of ruin. The balanced portfolio and TDF also have high ruin probabilities of 45.9% and 39.6%. When real returns on domestic stocks are poor over long investment periods, bonds and bills also tend to have poor real returns. Thus, the QDIAs provide little shelter during the storm.

Panel C of Figure C.1 examines the role of inflation. If realized inflation during retirement is low, the strategies perform relatively well with ruin probabilities ranging from 0.7% (optimal) to 6.0% (domestic stocks). If high inflation hits, the optimal strategy has a ruin probability of 15.9% versus 38.9% for domestic stocks, 51.1% for the balanced strategy, and 62.8% for the TDF. The low absolute correlation between inflation and international stock returns over long horizons (as shown in Table I) implies that international stocks provide crucial diversification benefits in inflationary periods.

The benefits of international diversification depend on the correlation between returns on domestic stocks and international stocks. This correlation varies over time [e.g., Longin and Solnik (2001)], so the value of international diversification could also vary. Panel D studies strategy performance conditional on the realized correlation between domestic and international stocks during the couple's retirement period. The ruin probability of the optimal strategy is stable across the quintiles, ranging from 4.6% to 7.7%. Other strategies actually have more dependence, with higher ruin probabilities when the realized domestic-international correlation is low. The low correlations seem to be proxying for worse economic times and wars when domestic markets do relatively poorly, and high correlations tend to line up with better asset class returns. Whatever the underlying causes may be, the results in Panel D assuage concerns that a high correlation between domestic and international stocks will invalidate the optimal age-based strategy.

C.3 Comparison of age-based and fixed-weight strategies

In Section 5.4, we demonstrate that the optimal fixed-weight allocation policy with 33% allocated to domestic stocks, 67% to international stocks, and 0% to bonds and bills provides nearly equivalent utility to the optimal age-based allocation policy. Table C.II compares the optimal fixed-weight and age-based strategies based on three retirement savings outcomes: the distribution of wealth at retirement (Panel A), the distribution of the income replacement rate (Panel B), and the distribution of wealth at death (Panel C). Table C.III compares the two strategies based on the distribution of the maximum drawdown during the working years (Panel A) and the distribution of the maximum drawdown during the retirement years (Panel B).

C.4 Welfare losses from deviating from the optimal asset allocation

In Section 5.4, we show that an optimal fixed-weight allocation of 33% in domestic stocks, 67% in international stocks, and 0% in bonds and bills achieves performance comparable with that of the optimal age-based strategy. The optimal age-based and fixed-weight investment strategies both deliver substantial welfare gains relative to the four benchmark strategies. We quantify below the welfare losses incurred by deviating from the optimal fixed-weight allocation.

Figure C.2 explores deviations from the optimal fixed-weight strategy and the associated equivalent savings rates. Panel A varies the allocation to domestic stocks between 0% and 100% within an all-equity design (i.e., the weights in bonds and bills are 0% and the remainder of the portfolio not invested in domestic stocks is allocated to international stocks). Expected utility as a function of the allocation to domestic stocks is relatively flat around the optimal allocation, and all allocations ranging from 11% domestic and 89% international to 55% domestic and 45% international have equivalent savings rates below 10.50%.

Figure C.2 also presents analogous results for adding bonds (Panel B) or bills (Panel C). In these analyses, the weight in bonds or bills ranges from 0% to 40% and the equity portion of the portfolio is split with relative weights of 33% in domestic stocks and 67% in international stocks. Our investors dislike even relatively small allocations to bonds. An allocation of 12% to bonds produces an 11.05% equivalent savings rate, which implies that the couples feel they need to increase their savings rate by more than 10% if they allocate 12% of their wealth to bonds. To achieve the same expected utility as saving 10.00% with the all-equity strategy, the couples must save 20% more to invest 20% in bonds, 35% more to invest 30%, and 54% more to invest 40%. The equivalent savings rates are even higher for allocations to bills in Panel C.

C.5 Endogenous retirement timing

In Section 5.4.2, we consider the optimal fixed-weight asset allocation policy for households under endogenous retirement timing. Table C.IV reports the estimated optimal retirement ages as a function of the household's current earnings, current retirement wealth, and expected Social Security benefit.

C.6 Alternative samples

In Section 5.4.4, we consider the optimal fixed-weight asset allocation policy estimated from samples restricted to countries with relatively large populations or large market capitalization-to-GDP ratios. Table C.V reports the sample start dates for developed countries in each of these alternative samples.

C.7 Leverage

In Section 5.4.4, we estimate the optimal fixed asset class weights and leverage levels for households who are allowed to borrow at a low (0.37% per year), medium (1.40%), or high (6.50%) spread above the local yield on short-term government bills. The results correspond to a household risk aversion parameter of $\gamma = 3.84$. Table C.VI reports the optimal asset allocation and leverage policies for alternative risk aversion parameters between 0.5 and 10.0.

C.8 American exceptionalism

In Section 5.4.5, we examine the role of American exceptionalism beliefs on optimal portfolio choice. Table C.VII reports the optimal allocation policy for investors under alternative subjective beliefs that forward-looking returns should be drawn from a distribution estimated from historical US returns rather than historical developed country returns.

Table C.I: Distribution of age at death

The table summarizes the distribution of age at death in years conditional on survival to age 25 based on the actuarial life tables from the SSA. For each investor type (i.e., heterosexual couple, single female, single male, female couple, or male couple), the table reports the mean, standard deviation, and distribution percentiles of the age at death. The statistics for the couples correspond to the age of the last survivor at death. The last column in the table shows the likelihood of death prior to reaching retirement age at 65.

	Moments		Percentiles									
Investor	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%	$\mathbb{E}[\mathbb{1}_{\{T_{max} \leq 480\}}]$
Female and male	87.6	9.1	59.4	70.8	76.1	83.0	88.9	93.8	97.7	100.0	104.5	0.023
Single female	81.9	13.9	36.0	54.2	62.7	75.5	84.9	91.5	96.3	98.9	103.8	0.119
Single male	77.2	15.3	30.6	46.1	55.9	68.9	80.6	88.3	93.5	96.2	101.1	0.195
Both female	89.3	8.5	62.7	74.0	78.7	84.9	90.3	94.9	98.8	101.1	105.5	0.014
Both male	85.5	9.7	55.9	67.2	72.8	80.6	87.0	92.1	96.1	98.4	102.8	0.038

Table C.II: Retirement saving outcomes: Age-based and fixed-weight asset allocation policies

The table summarizes the distributions of real wealth at retirement (Panel A), the real income replacement rate (Panel B), and real wealth at death (Panel C) across 1,000,000 bootstrap simulations for households adopting the optimal age-based and optimal fixed-weight asset allocation strategies. For each asset allocation strategy, the table reports the mean, standard deviation, and distribution percentiles of each measure of investment performance. Real wealth at retirement and real wealth at death are reported in millions of 2022 USD.

	Moments		Percentiles								
Strategy	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Panel A: Wealth at retirement ($MM)											
Optimal Age-Based	1.06	1.40	0.00	0.13	0.22	0.41	0.74	1.27	2.11	2.91	5.90
Optimal Fixed-Weight	1.07	1.43	0.00	0.13	0.22	0.41	0.74	1.27	2.11	2.91	5.94
Panel B: Income replacement rate											
Optimal Age-Based	1.24	1.55	0.00	0.59	0.68	0.83	1.05	1.39	1.92	2.44	4.54
Optimal Fixed-Weight	1.24	1.56	0.00	0.59	0.68	0.83	1.05	1.39	1.91	2.43	4.56
Panel C: Wealth at death ($MM)											
Optimal Age-Based	2.66	9.96	0.00	0.00	0.07	0.35	0.96	2.38	5.43	9.17	27.20
Optimal Fixed-Weight	2.94	11.25	0.00	0.00	0.07	0.37	1.03	2.59	5.99	10.18	30.54

Table C.III: Portfolio drawdowns: Age-based and fixed-weight asset allocation policies

The table summarizes the distributions of the maximum portfolio drawdown during the pre-retirement period (Panel A) and the maximum portfolio drawdown during the post-retirement period (Panel B) across 1,000,000 bootstrap simulations for households adopting the optimal age-based and optimal fixed-weight asset allocation strategies. For each asset allocation strategy and drawdown period, the table reports the mean, standard deviation, and distribution percentiles of the maximum portfolio drawdown.

	Moments		Percentiles								
Strategy	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Panel A: Working-period drawdown											
Optimal Age-Based	0.55	0.13	0.24	0.31	0.38	0.48	0.56	0.61	0.69	0.77	0.91
Optimal Fixed-Weight	0.55	0.12	0.24	0.31	0.38	0.48	0.56	0.61	0.68	0.76	0.91
Panel B: Retirement-period drawdown											
Optimal Age-Based	0.47	0.17	0.00	0.18	0.24	0.36	0.51	0.58	0.65	0.73	0.88
Optimal Fixed-Weight	0.48	0.17	0.00	0.19	0.25	0.38	0.52	0.59	0.66	0.74	0.88

Table C.IV: Optimal retirement ages

The table reports optimal retirement ages conditional on retirement balance level, income level, and Social Security level. At age 62, couples are divided into terciles in each of the three dimensions with independent sorts. The table shows the optimal retirement age for each of the 27 resulting couple types.

	Social Security level		
Income level	Low Social Security	Mid Social Security	High Social Security
Panel A: Low retirement account balance			
Low income	70	69	67
Mid income	70	69	67
High income	70	70	68
Panel B: Mid retirement account balance			
Low income	69	66	63
Mid income	69	67	65
High income	69	68	67
Panel C: High retirement account balance			
Low income	63	62	62
Mid income	65	64	62
High income	67	66	65

Table C.V: Country-level start dates for alternative samples.
The table shows the sample start dates for developed countries in the alternative samples with screens based on population or equity market size.

Country	Base sample	Country population $\geq$ 0.5% $\times$ world population	Market-cap/GDP ratio $\geq$ 0.5
United Kingdom	1890:01	1890:01	1890:01
Netherlands	1914:01	—	1919:01
Belgium	1897:01	—	1897:01
France	1890:01	1890:01	1890:01
Norway	1914:01	—	2006:01
Germany	1890:01	1890:01	1912:01
Denmark	1890:01	—	1890:01
Switzerland	1914:01	—	1928:01
United States	1890:01	1890:01	1928:01
Canada	1891:01	1933:01	1902:01
Argentina	1947:02	1947:02	—
New Zealand	1896:01	—	1900:01
Australia	1901:01	—	1932:01
Sweden	1910:01	—	1911:01
Austria	1920:01	—	1923:01
Chile period I	1927:01	—	1927:01
Greece	1981:02	—	1999:01
Czechoslovakia	1922:05	1922:05	—
Japan	1930:01	1930:01	1938:01
Portugal	1934:01	—	1999:01
Italy	1931:01	1931:01	2000:01
Ireland	1936:01	—	1945:01
Singapore	1998:07	—	1998:07
Iceland	2002:01	—	2002:01
Luxembourg	1982:01	—	1985:01
Türkiye	2010:02	2010:02	—
Spain	1959:01	1959:01	1998:01
Finland	1969:01	—	1998:01
Mexico	2001:08	2001:08	—
Czech Republic	2000:05	—	—
Hungary	1999:02	—	—
Poland	1999:06	1999:06	—
South Korea	2000:11	2000:11	2000:11
Slovakia	2000:01	—	—
Chile period II	2010:01	—	2010:01
Estonia	—	—	—
Israel	2010:01	—	2010:01
Slovenia	2010:01	—	—
Latvia	2016:01	—	—
Lithuania	2018:01	—	—
Colombia	2020:01	2020:01	2020:01

Table C.VI: Optimal fixed-weight asset allocation policies with leverage

The table reports the investment positions in domestic stocks, international stocks, bonds, and bills and the level of borrowing as a percentage of wealth for the optimal fixed-weight asset allocation policy under alternative risk aversion parameters.

	Optimal asset class weights				
Risk aversion	Domestic stocks	International stocks	Bonds	Bills	Borrowing (% of wealth)
Panel A: Borrowing spread of 6.50% (high)					
$\gamma = 0.5$	32%	68%	0%	0%	0%
$\gamma = 1.0$	35%	65%	0%	0%	0%
$\gamma = 2.0$	35%	65%	0%	0%	0%
$\gamma = 5.0$	33%	67%	0%	0%	0%
$\gamma = 7.5$	33%	67%	0%	0%	0%
$\gamma = 10.0$	33%	67%	0%	0%	0%
Panel B: Borrowing spread of 1.40% (medium)					
$\gamma = 0.5$	30%	70%	0%	0%	100%
$\gamma = 1.0$	35%	65%	0%	0%	100%
$\gamma = 2.0$	36%	64%	0%	0%	70%
$\gamma = 5.0$	34%	66%	0%	0%	50%
$\gamma = 7.5$	34%	66%	0%	0%	50%
$\gamma = 10.0$	34%	66%	0%	0%	55%
Panel C: Borrowing spread of 0.37% (low)					
$\gamma = 0.5$	30%	70%	0%	0%	100%
$\gamma = 1.0$	34%	66%	0%	0%	100%
$\gamma = 2.0$	31%	58%	11%	0%	100%
$\gamma = 5.0$	28%	56%	16%	0%	100%
$\gamma = 7.5$	28%	56%	16%	0%	100%
$\gamma = 10.0$	28%	57%	15%	0%	100%

Table C.VII: Optimal fixed-weight asset allocation policies under alternative subjective beliefs
The table reports the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal fixed-weight asset allocation policy under alternative subjective beliefs regarding the extent to which historical US returns and historical developed country returns reflect the forward-looking return distribution.

	Optimal asset class weights			
Probability US is "special" (x%)	Domestic stocks	International stocks	Bonds	Bills
0% (Developed sample only)	33%	67%	0%	0%
10%	38%	62%	0%	0%
20%	43%	57%	0%	0%
30%	48%	52%	0%	0%
40%	54%	46%	0%	0%
50%	60%	40%	0%	0%
60%	68%	32%	0%	0%
70%	76%	24%	0%	0%
80%	85%	15%	0%	0%
90%	96%	4%	0%	0%
100% (US sample only)	100%	0%	0%	0%

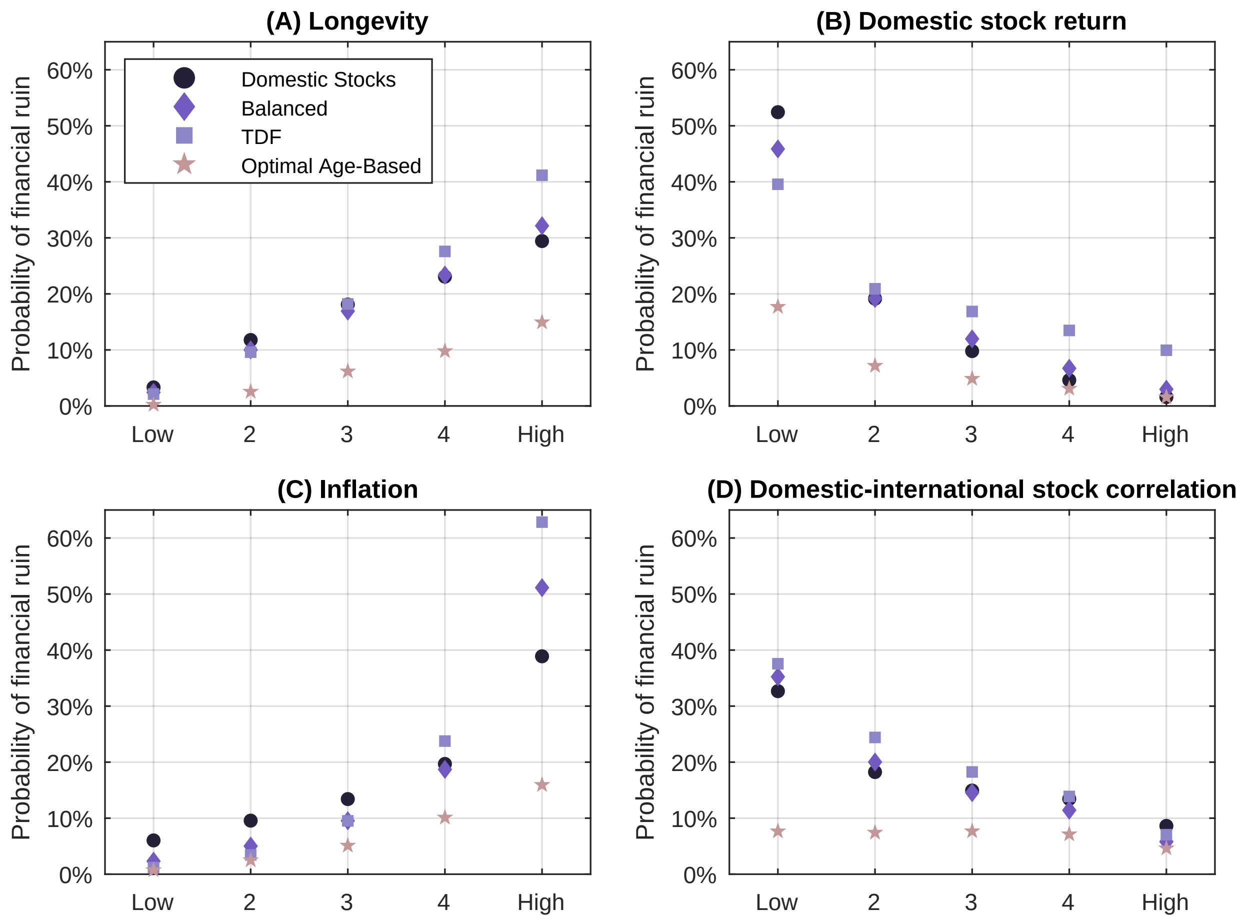

Figure C.1. Conditional ruin probabilities. The figure shows the probability of financial ruin conditional on quintile outcomes of household longevity (Panel A), realized returns for domestic stocks (Panel B), realized inflation (Panel C), and realized correlation between real returns for domestic stocks and international stocks (Panel D) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. The ruin probabilities in Panels B, C, and D condition on realizations during the retirement period.

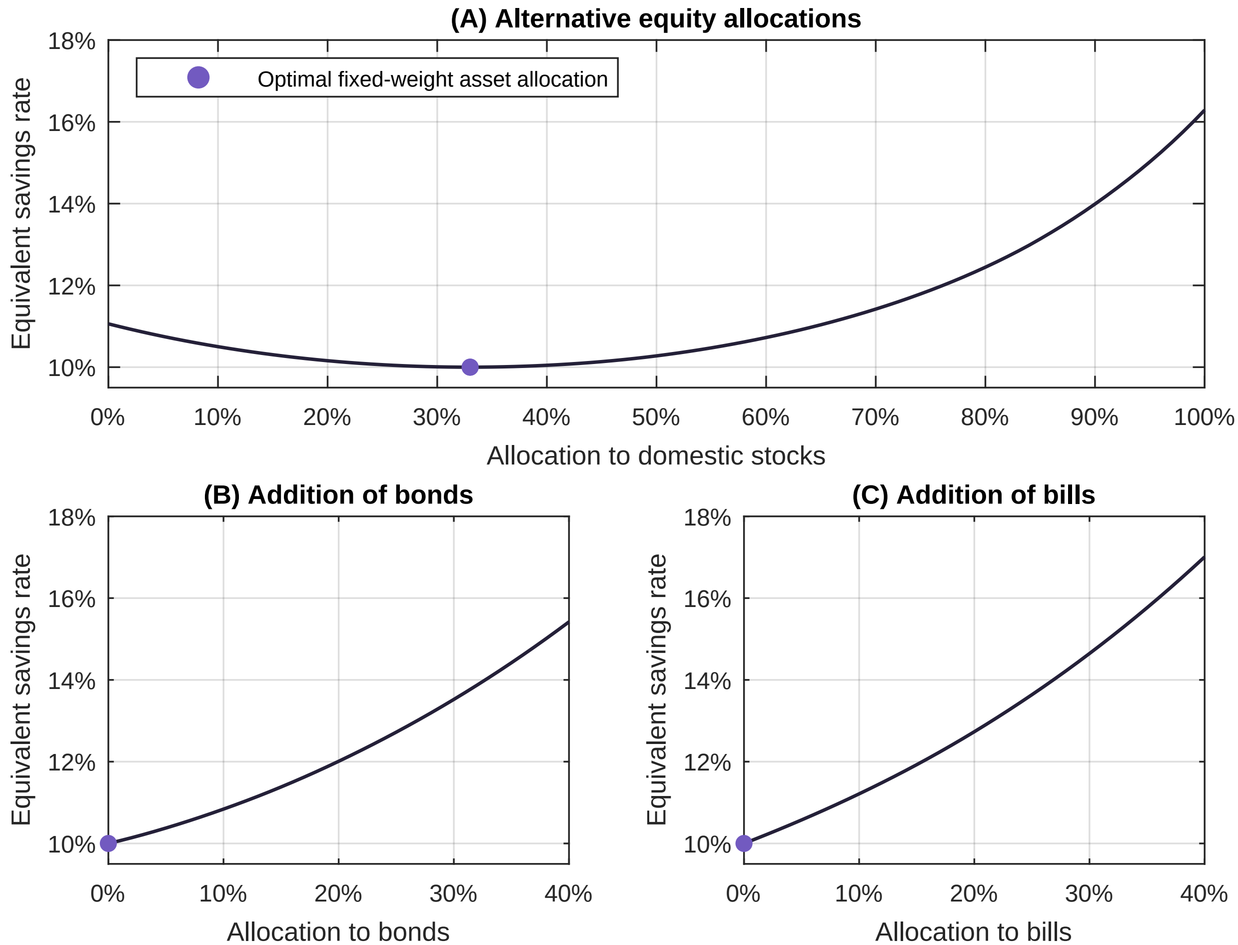

Figure C.2. Equivalent savings rates for deviations from the optimal fixed-weight portfolio. The figure shows equivalent savings rates to quantify relative economic value in pairwise comparisons of the optimal fixed-weight asset allocation strategy with alternative fixed-weight strategies. Each comparison is based on expected household utility over retirement consumption and bequest across 1,000,000 bootstrap simulations. In each plot, the base strategy corresponds to the optimal fixed-weight allocation of 33% in domestic stocks and 67% in international stocks with a pre-retirement period savings rate of 10%. The alternative strategies in Panel A adopt fixed-weight allocations to domestic stocks and international stocks, but deviate from the base strategy in weighting the two asset classes. The alternative strategies in Panel B (Panel C) adopt the same relative allocation to domestic stocks and international stocks as does the base strategy, but these strategies add a fixed allocation to bonds (bills). Each panel shows the household's equivalent savings rate for the indicated alternative strategy (i.e., the savings rate that equates the expected utility from retirement consumption and bequest for the alternative and base strategies).